Soft Bootstrap and Galileons

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From Scattering to Expansion, Northwestern University, 17th October 2019

Based on arXiv:1806.06079 and arXiv:1712.09937

H Elvang, M Hadjiantonis, C Jones, SP

Goals

- Understand effective field theories that result from spontaneous symmetry-breaking via on-shell recursion relations and soft behaviour of their amplitudes
- In particular, study classification, supersymmetrization and higher derivative corrections to Galileon theories

Spontaneous Symmetry-breaking

Off-shell: Vacuum isn't symmetric $\longrightarrow$ Broken symmetry is non-linearly realized $\longrightarrow$ Lagrangian fixed upto field redefintion and EOM

[Callan, Coleman, Wess, Zumino]

On-shell: Ward identity of broken current $\longrightarrow$ Soft behaviour of external Goldstone/Goldstino mode $\longrightarrow$ On-shell recursion for amplitudes

[Weinberg] [Cheung, Kampf, Novotny, Shen, Trnka]

Soft Bootstrap: Using on-shell recursion to constrain effective field theories

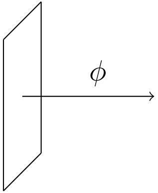
Applying the Soft Bootstrap

1. Classification of theories
2. Supersymmetrizations of known scalar theories
3. Higher derivative corrections and the double copy

Example: Galileon theories

Galileon Theories

- 3-brane in 5d:



$$S = \int \mathrm{d}^4x\sqrt{-G}\,\left[\Lambda_2^4 + \Lambda_3^3 K[G] + \Lambda_4^2 R[G] + \Lambda_5 \mathcal{K}_{\mathrm{GHY}}[G]\right]$$

DBI Cubic Quartic Quintic

- Galileon theory in decoupling limit $\Lambda_2 \to \infty$ with fixed

$$g_3 = \frac{\Lambda_3^3}{\Lambda_2^6},\ g_4 = \frac{\Lambda_4^2}{\Lambda_2^8},\ g_5 = \frac{\Lambda_5}{\Lambda_2^{10}},$$

- Also studied in: modified DGP gravity [Nicolis, Rattazzi, Trincherini]
  scalar decoupling limit of massive gravity [de Rahm, Gabadadze, Tolley]

Constraining Galileons Off-Shell

- Goldstone mode of spontaneously broken higher-dimensional Poincaré symmetry $\Rightarrow$ expect non-linear realization
  - ◇ Translation P_4
    $\Rightarrow$ Non-linearly realized as a shift symmetry $\phi \to \phi + c$
  - ◇ Lorentz rotations $M_{\mu 4}$
    $\Rightarrow$ Non-linearly realized as a spacetime-dependent shift symmetry
    $\phi \to \phi + c + v^\mu x_\mu + O(\phi)$

Higher-point interactions are fixed in terms of seed Galileon terms

$$\mathcal{L}_2 = [\partial^2 \phi^2]$$
$$\mathcal{L}_3 = [\partial^4 \phi^3]$$
$$\mathcal{L}_4 = [\partial^6 \phi^4]$$
$$\mathcal{L}_5 = [\partial^8 \phi^5]$$

- $\phi \to \phi + c$ is trivial
- $\phi \to \phi + c + v^\mu x_\mu + O(\phi)$ is non-trivial

Constraining Galileons On-shell

- Ward identity of broken current $\overset{!}{\Rightarrow}$ Adler zero
- Define soft weight σ_i,

$$\lim_{\epsilon \to 0} \mathcal{A}_n(p_1, \cdots, \epsilon p_i, \cdots, p_n) = O(\epsilon^{\sigma_i})$$

- Remain on-shell while taking soft limit [Elvang, Jones, Nachulich]
- In spinor helicity notation,

$$|i\rangle \to \epsilon |i\rangle \text{ for } h_i \geq 0$$
$$|i] \to \epsilon |i] \text{ for } h_i < 0$$

Because of spontaneously broken Lorentz symmetry, Galileons and DBI scalars have $\sigma = 2$

Quartic Galileon has enhanced $\sigma = 3 \Rightarrow$ special Galileon

Soft Subtracted Recursion

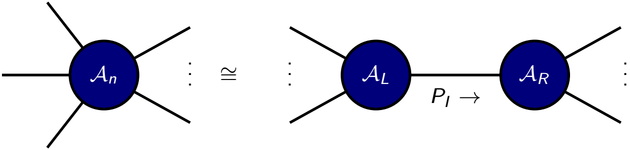
- Input: valence amplitudes, soft weights
- Soft shift:

$$p_i \to (1 - a_i z) p_i$$

- Recursion relation

$$\mathcal{A}_n(z=0) = \sum_I \operatorname*{Res}_{z=z_I^\pm} \frac{\mathcal{A}_L(z)\,\mathcal{A}_R(z)}{z\,P_I^2(z)\,\prod_i (1-a_i z)^{\sigma_i}}$$

[Cheung, Kampf, Novotny, Shen, Trnka]



Constructibility Criterion and Soft Bootstrap

- When is a theory completely fixed by soft behaviour?

$$4 - n - \frac{n-2}{v-2}[g_v] - \sum_i s_i - \sum_i \sigma_i < 0$$

  - v: valence of seed amplitude
  - $[g_v]$: mass dimension of seed coupling
  - s_i: spin *not* helicity

[Elvang, Hadjiantonis, Jones, SP]

- Do all constructible theories exist? No.
  Recursion can have spurious poles:

$$\mathcal{A}_n(z=0) = \sum_I \operatorname*{Res}_{z=z_I^\pm} \frac{\mathcal{A}_L(z)\,\mathcal{A}_R(z)}{z\,P_I^2(z)\prod_i (1-a_i z)^{\sigma_i}}$$

Locality $\Rightarrow$ No spurious poles $\Rightarrow$ Some on-shell data ruled out
$\Rightarrow$ Soft Bootstrap

Classifying Galileons

- Lagrangian construction is involved [Nicolis, Rattazzi, Trincherini]
- On shell, we have
  - Cubic Galileons: Field redefinition $\phi \to \phi + a(\partial\phi)^2$ gives

$$\tilde{g}_3\mathcal{L}_3 + \tilde{g}_4\mathcal{L}_4 + \tilde{g}_5\mathcal{L}_5 \to g_4\mathcal{L}_4 + g_5\mathcal{L}_5$$

  - Quartic Galileons: Uniquely fixed by $[g_4] = -6$, $v = 4$, $\sigma = 2, 3$

$$\sigma > 2 - \frac{2}{n} \Rightarrow \sigma \geq 2$$

  - Quintic Galileons: Uniquely fixed by $[g_5] = -9$, $v = 5$, $\sigma = 2$

Galileons are uniquely fixed by soft behaviour of their amplitudes!

Supersymmetrizing Galileons

- Previous work: [Koehn, Lehners, Ovrut] [Farakos, Germani, Kehagias]
- $\mathcal{N} = 1$ chiral multiplet consists of

$$\text{complex scalar: } Z, \bar{Z}$$
$$\text{fermion: } \psi, \bar{\psi}$$

- Supersymmetrization of $\mathcal{L}_4$ and $\mathcal{L}_5 \Rightarrow$ supersymmetrization of $\mathcal{L}_3$
- Is there a complex quartic Galileon $\mathcal{L}_4$?

$\sigma = 3$	$\times$
$\sigma = 2$	$\checkmark$

Recursing the $\mathcal{N} = 1$ Galileon

- SUSY Ward identities give all four-point amplitudes

$$\mathcal{A}_4(Z,\bar{Z},Z,\bar{Z}) = \frac{[24]}{[23]}\mathcal{A}_4(Z,\bar{Z},\psi,\bar{\psi}) = \frac{[24]}{[13]}\mathcal{A}_4(\psi,\bar{\psi},\psi,\bar{\psi})$$

- Recursible when $n_f \leq 2$

$\mathcal{A}_6(Z,\bar{Z},Z,\bar{Z},Z,\bar{Z}),\ \mathcal{A}_6(Z,\bar{Z},Z,\bar{Z},\psi,\bar{\psi})$ recursible ✓

$\mathcal{A}_6(Z,\bar{Z},\psi,\bar{\psi},\psi,\bar{\psi}),\ \mathcal{A}_6(\psi,\bar{\psi},\psi,\bar{\psi},\psi,\bar{\psi})$ SUSY Ward identities ✓

Evidence supports a $\mathcal{N} = 1$ quartic Galileon!

- While $\mathcal{A}_4$ and $\mathcal{A}_6$ match [Farakos, Germani, Kehagias], $\mathcal{A}_8$ is non-unique, i.e. not fixed by soft behaviour and supersymmetry.

$\mathcal{N} = 1$ Quintic Galileon?

- Complex quintic Galileon with $\sigma = 2$? ✓
- General ansatz for $\mathcal{A}_5(Z, \bar{Z}, Z, \bar{Z}, Z)$ with
  - ◇ Little group $SO(2)$
  - ◇ $[g_5] = -9$
  - ◇ Bose symmetry
  - ◇ $\sigma = 2$
- SUSY Ward identity gives

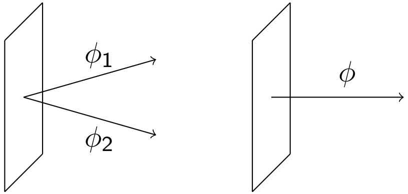
$$\mathcal{A}_5(Z, \bar{Z}, Z, \bar{\psi}, \psi) = -\frac{[25]}{[24]}\mathcal{A}_5(Z, \bar{Z}, Z, \bar{Z}, Z)$$

  must be local.
- No choice gives local $\mathcal{A}_5(Z, \bar{Z}, Z, \bar{\psi}, \psi)$

No $\mathcal{N} = 1$ quintic Galileon with $\sigma = 2$ complex scalar!
No $\mathcal{N} = 1$ cubic Galileon with $\sigma = 2$ complex scalar!

Coupling the Quintic Galileon to an R-Goldstone

- Re(Z) has $\sigma = 2$ and Im(Z) has $\sigma = 1$: recursing these amplitudes to 7 and 8 point gives you consistent results



- Consider a 3-brane in $5d$ space
- One broken Lorentz generator $\Rightarrow$ one $\sigma = 2$ Goldstone boson
- Goldstone mode arising from R-symmetry breaking, e.g. $U(1)_R$ will have $\sigma = 1$

Evidence of $\mathcal{N} = 1$ quintic Galileon coupled to an R-Goldstone!

Higher Derivative Corrections to Special Galileon

(without supersymmetry)

- Constructibility for $\sigma = 3$ real scalar: $-14 < [g_4] \leq -6$
- Bose symmetric ansatz for $\mathcal{A}_4(\phi,\phi,\phi,\phi)$ recursible to $\mathcal{A}_6$

[Gonzalez, Penco, Trodden]

$$\underset{[\partial^6\phi^4]}{\frac{c_0}{\Lambda^6} stu} + \underset{[\partial^{10}\phi^4]}{\frac{c_1}{\Lambda^{10}}(s^5+t^5+u^5)} + \underset{[\partial^{12}\phi^4]}{\frac{c_2}{\Lambda^{12}} s^2t^2u^2 + \frac{c_3}{\Lambda^{12}}(s^6+t^6+u^6)}$$

- Similarly, we can construct an ansatz for Bose symmetric $\mathcal{A}_5(\phi,\phi,\phi,\phi,\phi)$ that is recursible to $\mathcal{A}_7$ and $\mathcal{A}_8$

Higher derivative contributions have a fixed form!

Double Copy Construction of Special Galileon

$$\underbrace{\text{Chiral perturbation theory}}_{O(\Lambda^{-2})} \otimes \underbrace{\text{Chiral perturbation theory}}_{O(\Lambda^{-4})} \stackrel{?}{=} \underbrace{\text{Special Galileon}}_{O(\Lambda^{-8})}$$

$$1 \otimes 1 = 3$$

[Cachazo, He, Yuan]

- Construct χPT partial amplitude that is color-ordered and BCJ compatible
- Input this ansatz into KLT relations...

Double copy constructions works for $\mathcal{A}_4$ and $\mathcal{A}_6$
Doesn't work for $\mathcal{A}_5$!

TL; DR

- Generically, spontaneous symmetry-breaking leads to vanishing soft limits on-shell
- We exploit this property to recurse amplitudes
- Soft Bootstrap: Locality of recursed amplitudes allows us to rule out theories with certain on-shell data (σ_i, v, $[g_v]$, s_i)
- Applications:
  - ◇ Classification: Galileons are uniquely fixed by soft behaviour, complex special Galileon doesn't exist
  - ◇ Supersymmetrization: $\mathcal{N} = 1$ quartic and quintic + R-Goldstone exist
  - ◇ Higher derivative correction: Double copy construction of the special Galileon works for even-point

The Effort Continues...

- Expanding color structure
  [Low, Yin]
- Three-point interactions
- Fixing EFTs arising from massive gravity
  [Bonifacio, Hinterbichler, Johnson, Joyce]
- Helicity-dependent vector soft shifts
  [Cheung, Kampf, Novotny, Shen, Trnka, Wen]
- ... and more!

Thank you!