Generalizing the Double Copy: The KLT Bootstrap

Shruti Paranjape

University of Michigan, Ann Arbor
Leinweber Center for Theoretical Physics

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H-H Chi, A Herderschee, H Elvang, C R T Jones, SP
L Johnson, C R T Jones, SP

Double Copy

The double copy is a map between amplitudes in

$$\text{Theory C} \;=\; \text{Theory A} \;\otimes\; \text{Theory B}$$

More precisely, for adjoint particles

$$\mathcal{M}_n^{\mathsf{C}} = \sum_{\substack{\alpha\in\mathcal{B}_A \\ \beta\in\mathcal{B}_B}} \mathcal{A}_n^A[\alpha]\, S_{\mathsf{KLT}}[\alpha|\beta]\, \mathcal{A}_n^B[\beta]\,.$$

Original example:

$$\text{Open} \;\otimes_{\mathsf{KLT}}\; \text{Open} \;=\; \text{Closed} \;\xrightarrow{\alpha'\to 0}\; \text{YM} \;\otimes_{\mathsf{KLT}}\; \text{YM} \;=\; \text{GR}+\phi$$

[Kawai, Lewellen, Tye]

Identity of the KLT Algebra

This has been extended to many theories, e.g. massless adjoint single-copies,

	ϕ^3	χPT	YM
ϕ^3	ϕ^3	χPT	YM
χPT	χPT	Special Galileon	Born-Infeld
YM	YM	Born-Infeld	Dilaton Gravity

[Cachazo, He, Yuan]

Zeroth copy: ϕ^3 bi-adjoint scalar theory acts as the identity

$$\phi^3 \otimes \mathsf{A} = \mathsf{A}$$
$$\mathsf{A} \otimes \phi^3 = \mathsf{A}$$
$$\phi^3 \otimes \phi^3 = \phi^3$$

Using the Identity

Since $\phi^3 \otimes \phi^3 = \phi^3$,

$$m[\alpha|\beta]S_{\text{KLT}}[\beta|\gamma]m[\gamma|\beta] = m[\alpha|\beta]$$
$$\Rightarrow S_{\text{KLT}}[\beta|\gamma] = m[\gamma|\beta]^{-1} = m^{-1}[\beta|\gamma]$$

$\Rightarrow$ the KLT kernel is determined as the inverse of a full-rank zeroth copy matrix of amplitudes.

The single copy theories must satisfy:

- Right BCJ relations: $\phi^3 \otimes \text{A} = \text{A}$
- Left BCJ relations: $\text{A} \otimes \phi^3 = \text{A}$

These give basis-independence of the KLT formula on the right (left),

$$m[\alpha|\beta]S_{\text{KLT}}[\beta|\gamma]\mathcal{A}[\gamma] = m[\alpha|\beta]S_{\text{KLT}}[\beta|\delta]\mathcal{A}[\delta]$$

[Bern, Carrasco, Johansson]

Goals

For every choice of zeroth copy, we can define a KLT double copy map!

Generalize the field theory KLT formula to include:

1. Massive States ✓

[Johnson, Jones, SP]

2. Different color structures
3. All orders in the derivative/EFT expansion

[Carrasco, Rodina, Yin, Zekioglu]

[Chi, Elvang, Herderschee, Jones, SP]

Different Color Structures at Leading Order

There are many bi-colored scalar theories to choose as zeroth copy:

- Usual bi-adjoint scalar theory: $g\, f^{abc} f^{a'b'c'} \phi_{aa'}\phi_{bb'}\phi_{cc'}$
- 3-point: $\lambda_3\, d^{abc} d^{a'b'c'} \phi_{aa'}\phi_{bb'}\phi_{cc'}$
- 4-point: $\lambda_4\, d^{abcd} d^{a'b'c'd'} \phi_{aa'}\phi_{bb'}\phi_{cc'}\phi_{dd'}$

$$\vdots$$

where $d^{a_1 a_2 \cdots a_n} = \mathrm{Tr}\left[T^{a_1} T^{(a_2} \cdots T^{a_n)}\right]$.

Example: 3-Point Modifications

Choose a generic leading order 3-point interaction,

$$m_3[123|123] = g + \lambda_3 \,,$$
$$m_3[123|132] = -g + \lambda_3 \,,$$

Putting these together,

$$m_4[1234|1234] = (g+\lambda_3)^2 \left(\frac{1}{s} + \frac{1}{u}\right) \,,$$
$$m_4[1234|1243] = (-g^2+\lambda_3^2)\ \frac{1}{s} \,,$$
$$m_4[1234|1432] = (-g+\lambda_3)^2 \left(\frac{1}{s} + \frac{1}{u}\right) \,.$$

KLT kernel produces local double copies at 4-point for generic g and λ_3.

Locality at 5-Point

Couplings	Matrix Rank	Spurious Singularities?
$g \neq 0,\ \lambda_3 \neq 0$	24	Yes
$g \neq 0,\ \lambda_3 = \sqrt{3}g$	21	Yes
$g = 0,\ \lambda_3 \neq 0$	11	Yes
$g \neq 0,\ \lambda_3 = 0$	2	No

We have two options:

- BCJ relations $\not\Rightarrow$ locality: Spurious singularities are canceled, e.g. $d^{abc}\phi_a\phi_b\phi_c$ double copies to ϕ^3
- BCJ relations $\Rightarrow$ locality: Only consider the usual bi-adjoint scalar theory at 3-point at leading order

Minimal Rank at Leading Order

- Requiring that a generic $S_{\text{KLT}}[\alpha|\beta] = m^{-1}[\alpha|\beta]$ has $(n-3)!$ minimal rank ensures there are no spurious singularities.
- If the rank is non-minimal, spurious singularities appear in the kernel.

Independent bi-adjoint scalar amplitudes:

$$
\begin{array}{c}
n! \times n! \\
\Big\downarrow \text{Cyclicity} \\
(n-1)! \times (n-1)! \\
\Big\downarrow \text{KK/monodromy/? relations} \\
(n-2)! \times (n-2)! \\
\Big\downarrow \text{Self-copy conditions} \\
(n-3)! \times (n-3)!
\end{array}
$$

Minimal Rank for EFTs

Interested in zeroth-, single- and double-copy theories corrected by higher dimension operators,

$$m_n[\beta|\alpha] = m_n^{(0)}[\beta|\alpha] + m_n^{(1)}[\beta|\alpha] + \ldots \ ,$$
$$\mathcal{A}_n[\alpha] = \mathcal{A}_n^{(0)}[\alpha] + \mathcal{A}_n^{(1)}[\alpha] + \ldots \ .$$

Assuming UV physics decouples when $\Lambda \to \infty$,

$$\lim_{\Lambda\to\infty} m_n[\beta|\alpha] = m_n^{(0)}[\beta|\alpha], \quad \lim_{\Lambda\to\infty} \mathcal{A}_n[\alpha] = \mathcal{A}_n^{(0)}[\alpha] \ , \quad \lim_{\Lambda\to\infty} \mathcal{M}_n = \mathcal{M}_n^{(0)}$$

$\Rightarrow$ Higher-derivative corrections can not change the leading order rank.

Allowing Different Color Structures

At leading order, only one color structure $f^{abc} \Rightarrow$ Kleiss-Kuijf relations,

$$m_n[1, \alpha, n, \beta|\gamma] = (-1)^{|\beta|} \sum_{\sigma \in \alpha \sqcup\!\sqcup \beta} m[1, \sigma, n|\gamma]$$

Higher-derivative corrections to bi-adjoint scalar theory may have different color structures e.g.

$$d^{abc} = \mathrm{Tr}\left[T^a T^{(b} T^{c)}\right] ,$$
$$d^{abcd} = \mathrm{Tr}\left[T^a T^{(b} T^c T^{d)}\right] , \dots$$

To account for different color structures, we start with $(n-1)!$ basis rather than an $(n-2)!$ KK-independent basis $\rightarrow$ and then we impose $(n-3)!$ minimal rank.

The KLT Bootstrap

Choose a bi-colored scalar theory with rank $(n-3)!$ as zeroth copy

$\downarrow$

Find single-copy amplitudes compatible with the extended BCJ relations

$\downarrow$

Calculate KLT double-copy amplitudes

Step 1: 4-Point Kernel

- Start with a 6×6 matrix $m[\alpha|\beta]$,

$$\begin{aligned}
m_4[1234|1234] &= f_1(s,t) = f_1(-s-t,t)\,,\\
m_4[1234|1243] &= f_2(s,t)\,,\\
m_4[1234|1324] &= f_3(s,t) = f_2(-s-t,t)\,,\\
m_4[1234|1342] &= f_4(s,t) = f_2(s,t)\,,\\
m_4[1234|1423] &= f_5(s,t) = f_2(-s-t,t)\,,\\
m_4[1234|1432] &= f_6(s,t) = f_6(-s-t,t)\,,
\end{aligned}$$

- Impose bootstrap condition BAS $\times$ BAS $=$ BAS on the three independent functions,

$$\boxed{\begin{gathered}
f_6(s,t) = f_1(s,t)\\
f_1(s,t) = \frac{f_2(s,t)f_2(-s-t,s)}{f_2(t,s)}\\
f_2(s,t)f_2(-s-t,s)f_2(t,-s-t) = f_2(t,s)f_2(-s-t,t)f_2(s,-s-t)
\end{gathered}}$$

[Chi, Herderschee, Elvang, Jones, SP]

Comparing to String Theory

Can the bootstrap equations be solved?

Known solution: String theory

$$f_2(s,t) = \frac{1}{\sin(\alpha' \pi s)}$$

$$\Rightarrow m[1234|1243] = \frac{1}{\pi \alpha' s} + \frac{\pi \alpha' s}{6} + \frac{7}{360}\pi^3 \alpha'^3 s^3 + O\left(\alpha'^4\right)$$

$$\Rightarrow S[1234|1243] = \sin(\alpha' \pi s) = \pi \alpha' s - \frac{1}{6}\alpha'^3 \left(\pi^3 s^3\right) + O\left(\alpha'^4\right)$$

[Bjerrum-Bohr, Damgaard, Søndergaard][Mizera]

Another possible solution

$$f_2(s,t) = \frac{1}{s}\frac{G_1(s) G_2(t)}{G_3(u)}$$

Perturbative Solution to Bootstrap Equation

- Begin with an ansatz for $f_2(s,t)$,

$$f_2(s,t) = -\frac{g^2\Lambda^2}{s} + \sum_{k=0}^{N}\sum_{r=0,k} \frac{a_{k,r}}{\Lambda^{2k}}\, s^r\, t^{k-r}$$

- Solve bootstrap equations,

$$\begin{aligned} f_2(s,t) = &-\frac{g^2\Lambda^2}{s} + \frac{1}{\Lambda^2}(a_{1,0}t + a_{1,1}s) + \frac{a_{2,0}}{\Lambda^4}t(s+t) \\ &+ \frac{1}{\Lambda^6}\left[a_{3,0}t^3 + a_{3,1}st^2 + a_{3,2}s^2t + a_{3,3}s^3\right] + \mathcal{O}\left(\frac{1}{\Lambda^8}\right) \end{aligned}$$

- To recover string theory result,

$$a_{k,i} = 0 \text{ for } k > i \qquad a_{1,1} = -\frac{1}{6}\,, \quad a_{3,3} = -\frac{7}{360}\,, \ldots$$

- As expected of bottom-up approach, this solution is more general

[Chi, Herderschee, Elvang, Jones, SP]

Zeroth Copy Lagrangian

$$
\begin{aligned}
\mathcal{L} = & - \frac{1}{2} (\partial\phi)^2 + f^{abc} f^{a'b'c'} \phi^{aa'} \phi^{bb'} \phi^{cc'} \\
& + \frac{a_L + a_R}{2\Lambda^4} f^{abx} f^{cdx} f^{a'b'x'} f^{c'd'x'} (\partial_\mu \phi^{aa'})(\partial^\mu \phi^{bb'}) \phi^{cc'} \phi^{dd'} \\
& + \frac{a_R}{\Lambda^4} f^{abx} f^{cdx} d^{a'b'x'} d^{c'd'x'} (\partial_\mu \phi^{aa'}) \phi^{bb'} (\partial^\mu \phi^{cc'}) \phi^{dd'} \\
& + \frac{a_L}{\Lambda^4} d^{abx} d^{cdx} f^{a'b'x'} f^{c'd'x'} (\partial_\mu \phi^{aa'}) \phi^{bb'} (\partial^\mu \phi^{cc'}) \phi^{dd'} \ .
\end{aligned}
$$

Step 2: Single-copy at 3-Point

What higher derivative corrections can we add to YM so that $(\text{YM} + \text{hd}) \otimes_{\text{KLT}} (\text{YM} + \text{hd}) = (\text{GR} + \text{hd})$?

$\Rightarrow$ Corrections must satisfy the BCJ relations i.e. BAS $\times$ YM $=$ YM.

At 3-point this amounts to anti-symmetry of identical states, so the possible interactions are

$$\mathcal{A}_3[1^-, 2^-, 3^+] = g_{\text{YM}} \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 31 \rangle}$$

$$\mathcal{A}_3[1^-, 2^-, 3^-] = \frac{g_{F^3}}{\Lambda^2} \langle 12 \rangle \langle 23 \rangle \langle 31 \rangle$$

YM at 4-Point

A local ansatz for the 4-point Yang-Mills amplitude is then,

$$\mathcal{A}_4[1^+2^+3^-4^-] = \frac{[12]^2\langle 34\rangle^2}{su}\left[\left(g_{\text{YM}}^2 - \frac{1}{\Lambda^4}g_{F^3}^2 ut\right) + \sum_{k=2}^{N}\sum_{r=1}^{k-1}\Lambda^{-2k}e_{k,r}s^r u^{k-r}\right]$$

The final answer is

$$\mathcal{A}_4^L[1^+2^+3^-4^-] = [12]^2\langle 34\rangle^2\left[\frac{(g_{\text{YM}}^L)^2}{su} + \frac{1}{\Lambda^4}\left(\frac{(g_{\text{YM}}^L)^2}{g^2}(a_{1,1} - a_{1,0}) - (g_{F^3}^L)^2\frac{t}{s}\right) - \frac{e_{3,1}^L}{\Lambda^6}t + \mathcal{O}\left(\frac{1}{\Lambda^8}\right)\right].$$

[Chi, Herderschee, Elvang, Jones, SP]

Step 3: Corrections to Gravity

Double-copying the YM EFT amplitudes with themselves give us a gravity EFT amplitude,

$$\begin{aligned}\mathcal{M}_4(1^+2^+3^-4^-) =& [12]^4\langle 34\rangle^4 \\ &\times\Bigg[-\frac{(g^L_{\mathrm{YM}})^2(g^R_{\mathrm{YM}})^2}{g^2\Lambda^2}\frac{1}{stu}+\frac{((g^L_{\mathrm{YM}})^2(g^R_{F^3})^2+(g^R_{\mathrm{YM}})^2(g^L_{F^3})^2)}{g^2\Lambda^6}\frac{1}{s} \\ &+\frac{1}{\Lambda^8}\left(\frac{(g^L_{\mathrm{YM}})^2(g^R_{\mathrm{YM}})^2}{g^4}a_{2,0}+\frac{1}{g^2}((g^R_{\mathrm{YM}})^2e^L_{3,1}+(g^L_{\mathrm{YM}})^2e^R_{3,1})\right) \\ &+\mathcal{O}\left(\frac{1}{\Lambda^{10}}\right)\Bigg]\end{aligned}$$

- Single-copy (YM) : Larger class of models can be double-copied using the new KLT kernel
- Double-copy (GR) : Same corrections are produced regardless of which KLT kernel is used

[Chi, Herderschee, Elvang, Jones, SP]

Summary of 4-Point Results

Schematic Operator	Total	Generalized	String	Cubic BAS
$\mathrm{Tr}[F^4]$	1	1	0	$\times$
$\mathrm{Tr}[D^2 F^4]$	2	1	1	1
$\mathrm{Tr}[D^4 F^4]$	3	3	1	1
$\mathrm{Tr}[D^6 F^4]$	4	3	2	2

Schematic Operator	Total	Generalized	Cubic BAS
R^4	1	1	1
$\nabla^2 R^4$	1	1	1
$\nabla^4 R^4$	2	2	2

Higher-Derivative KLT

- Local extensions of the double copy can be formulated via deformations of bi-adjoint scalar theory that satisfy the minimal rank condition
- For massive deformations, this is imposed by the spectral conditions satisfied by KK towers and Compton scattering
- For effective field theories, the KLT bootstrap equations may be solved to derive general BCJ-compatible corrections to YM

Outlook and Open Questions

- Landscape exploration: What other discontinuous deformations satisfy the minimal rank conditions?
- Different particles: How does this work for particles not in the adjoint representation?

  [Brown, Naculich][Huang, Johansson, Lee][Johansson, Mogull, Teng][Huang, Johansson]
- BCJ and CHY: Is our prescription related to color-kinematics duality and CHY?

  [Carrasco, Rodina, Yin, Zekioglu][Cachazo, He, Yuan]

Thank you