



Locality Constraints on a Massive Double Copy

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Based on [arXiv:2004.12948](https://arxiv.org/abs/2004.12948)

L Johnson, C Jones, SP

Goals

- Understand factorization, decoupling and locality of a naive massive extension of the BCJ double copy
- Does massive Yang-Mills double copy to massive gravity?

[Johansson, Ochirov, Naculich, Chiodaroli, Gunaydin, Roiban, Bautista, Guevara...]

Massless BCJ Color-Kinematics

[Bern, Carrasco, Johansson]

$$\mathcal{A}_n = \sum_i \frac{c_i n_i}{p_i} \rightarrow \mathcal{M}_n = \sum_i \frac{\tilde{n}_i n_i}{p_i}$$

Numerators are not unique because of generalized gauge transformations.

Ex: At 4-point,

$$n_s + n_t + n_u = 0 \text{ and } \mathcal{A}_4 = \frac{c_s n_s}{s} + \frac{c_t n_t}{t} + \frac{c_u n_u}{u}$$

are invariant under

$$n_s \rightarrow n_s + s \Delta$$

$$n_t \rightarrow n_t + t \Delta$$

$$n_u \rightarrow n_u + u \Delta$$

$\Rightarrow$ one function's worth of color-kinematics-satisfying numerators.

Massive BCJ Color-Kinematics

[Johnson, Jones, SP]

$$\mathcal{A}_4 = \frac{c_s n_s}{s + m^2} + \frac{c_t n_t}{t + m^2} + \frac{c_u n_u}{u + m^2}$$

This is invariant under a generalized gauge transformation,

$$n_s \rightarrow n_s + (s + m^2)\Delta$$

$$n_t \rightarrow n_t + (t + m^2)\Delta$$

$$n_u \rightarrow n_u + (u + m^2)\Delta$$

BUT kinematic Jacobi identity is **not** preserved

$$\tilde{n}_s + \tilde{n}_t + \tilde{n}_u = (n_s + n_t + n_u) + (s + t + u + 3m^2)\Delta = -m^2\Delta.$$

$\Rightarrow$ there is **no** generalized gauge freedom.

A Corollary

Start with numerators in a massive theory that do not satisfy kinematic Jacobi,

$$n_s + n_t + n_u = \Delta$$

Performing a generalized gauge transformation

$$n_s \rightarrow n_s + (s + m^2) \frac{\Delta}{m^2}$$

$$n_t \rightarrow n_t + (t + m^2) \frac{\Delta}{m^2}$$

$$n_u \rightarrow n_u + (u + m^2) \frac{\Delta}{m^2}$$

results in

$$\tilde{n}_s + \tilde{n}_t + \tilde{n}_u = (n_s + n_t + n_u) + (s + t + u + 3m^2) \frac{\Delta}{m^2} = 0.$$

$\Rightarrow$ Color-kinematics is **trivial** for massive theories at 4-point.

Finding Numerators in Massive Theories

- Systematic way of finding the **unique** set of numerators

$$\begin{bmatrix} \mathcal{A}_4[1234] \\ \mathcal{A}_4[1324] \end{bmatrix} = \begin{bmatrix} \frac{1}{s+m^2} + \frac{1}{u+m^2} & \frac{1}{u+m^2} \\ -\frac{1}{u+m^2} & \frac{1}{t+m^2} - \frac{1}{u+m^2} \end{bmatrix} \begin{bmatrix} n_s \\ n_t \end{bmatrix}$$

$$\begin{bmatrix} n_s \\ n_t \end{bmatrix} = \begin{bmatrix} \frac{(m^2+s)(2m^2+s)}{m^2} & -\frac{(m^2+s)(m^2+t)}{m^2} \\ \frac{(m^2+s)(m^2+t)}{m^2} & -\frac{(m^2+t)(2m^2+t)}{m^2} \end{bmatrix} \begin{bmatrix} \mathcal{A}_4[1234] \\ \mathcal{A}_4[1324] \end{bmatrix}$$

- Matrix corresponds to **massive bi-adjoint scalar** amplitudes
- Matrix is **invertable** $\Rightarrow$ we can solve for the numerators uniquely
- No null vectors $\Rightarrow$ **No BCJ relations**
- Kinematic Jacobi-satisfying numerators exist for **any** massive theory
- No new poles are generated at 4-point

Massless or Decoupling Limit

Naive $m \rightarrow 0$ type massless limits are **ill-defined** because of loss of dofs.
Simple example: a massive photon

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu + g(A_\mu A^\mu)^2$$

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Stückelberg trick:

$$A_\mu \rightarrow A_\mu + \partial_\mu \phi$$

$$\begin{aligned} \delta\mathcal{L} = & -mA_\mu\partial^\mu\phi A^\mu - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + 4\frac{g}{m}A_\mu A^\mu A_\nu\partial^\nu\phi + \frac{g}{m^4}(\partial_\mu\phi\partial^\mu\phi)^2 \\ & + \frac{g}{m^2}\left(4(A_\mu\partial^\mu\phi)^2 + 2A_\mu A^\mu\partial_\nu\phi\partial^\nu\phi\right) + 4\frac{g}{m^3}A_\mu\partial^\mu\phi\partial_\nu\phi\partial^\nu\phi. \end{aligned}$$

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Only one non-singular limit:

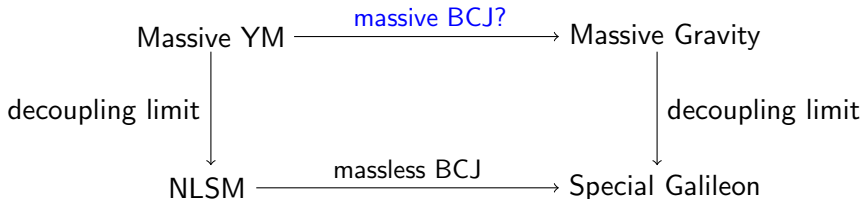
$$g, m \rightarrow 0, \quad \Lambda = \left(\frac{m^4}{g}\right)^{1/4}, \quad A_\mu, \phi \text{ fixed},$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{\Lambda^4}(\partial_\mu\phi\partial^\mu\phi)^2$$

Example: Massive Yang-Mills

$$\begin{aligned}\mathcal{L}_{\text{mYM}} &= -\frac{1}{4} \left(\partial_{[\mu} A_{\nu]}^a \right)^2 - \frac{1}{2} m^2 A_\mu^a A^{a\mu} - g f^{abc} A_\mu^a A_\nu^b \partial^\mu A^{c\nu} - \frac{1}{4} g^2 f^{abe} f^{cde} A_\mu^a A^{\mu c} A_\nu^b A^{\nu d} \\ &= \mathcal{L}_{\text{YM}} - \frac{1}{2} m^2 A_\mu^a A^{a\mu}\end{aligned}$$

Goldstone boson equivalence theorem: Longitudinal modes decouple to give **NLSM pions** in the high energy/massless/decoupling limit.



Can we construct massive gravity from massive YM?

Double-Copying Massive Yang-Mills

Strategy for constructing BCJ form of the amplitude:

Feynman rules $\longrightarrow$ Partial amplitude $\longrightarrow$ BCJ numerators

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Feynman rules $\longrightarrow$ Partial amplitude $\longrightarrow$ BCJ numerators

$$A^\mu \otimes A^\nu = 3 \otimes 3 = \textcolor{red}{5} \oplus \textcolor{green}{3} \oplus \textcolor{blue}{1} = \textcolor{red}{h}^{\mu\nu} \oplus \textcolor{green}{B}^{\mu\nu} \oplus \textcolor{blue}{\phi}.$$

- Build projection operators
- Calculate amplitudes with different external states
- Use these to check factorization of graviton amplitudes at higher-point

Massive Gravity as a Double Copy: 3-point

At 3-point,

$$\mathcal{A}_3 = 2g \left((\epsilon_2 \cdot \epsilon_3)(\epsilon_1 \cdot p_2) + (\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot p_3) + (\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot p_1) \right).$$
$$\mathcal{M}_3 = \frac{2}{M_p} \left((\epsilon_2 \cdot \epsilon_3)(\epsilon_1 \cdot p_2) + (\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot p_3) + (\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot p_1) \right)^2.$$

where $\epsilon_i^{\mu\nu} = \epsilon_i^\mu \epsilon_i^\nu$ is replaced by projection operator for different states:

- Dilaton parity is **violated** so graviton amplitudes have dilaton channels.
- B -parity is **preserved**.

Massive Gravity as a Double Copy: 4-point

At 4-point,

$$\begin{aligned} n_s = & [(\epsilon_1 \cdot \epsilon_2) p_1^\mu + 2(\epsilon_1 \cdot p_2) \epsilon_2^\mu - (1 \leftrightarrow 2)] \left(g_{\mu\nu} + \frac{(-p_{1\mu} - p_{2\mu})(p_{3\nu} + p_{4\nu})}{m^2} \right) \\ & \times [(\epsilon_3 \cdot \epsilon_4) p_3^\nu + 2(\epsilon_3 \cdot p_4) \epsilon_4^\nu - (3 \leftrightarrow 4)] \\ & + (s + m^2) [(\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot \epsilon_4) - (\epsilon_1 \cdot \epsilon_4)(\epsilon_2 \cdot \epsilon_3)], \end{aligned}$$

with $n_t = n_s|_{1 \rightarrow 3 \rightarrow 2 \rightarrow 1}$ and $n_u = n_s|_{1 \rightarrow 2 \rightarrow 3 \rightarrow 1}$.

$$\mathcal{M}_4 = \frac{n_s^2}{s + m^2} + \frac{n_t^2}{t + m^2} + \frac{n_u^2}{u + m^2}$$

- Factorizes into $\mathcal{A}_3(hh\phi)$ and $\mathcal{A}_3(hhh)$ correctly
- Matches well-known dRGT theory of ghost-free massive gravity.

[deRham, Gabadadze, Tolley]

[Momeni, Rumbutis, Tolley]

Pions and Special Galileons

$$\mathcal{A}_4(g_L g_L g_L g_L) \xrightarrow{\text{decouple}} \mathcal{A}_4(\pi \pi \pi \pi)$$

IF massive double copy and decoupling limit commute, we expect

$$\mathcal{M}_4(h_L h_L h_L h_L) \xrightarrow{\text{decouple}} \mathcal{M}_4(\phi \phi \phi \phi)$$

to be the only non-zero contribution. We find

$$\mathcal{M}(0000) = \frac{7}{144} st(s+t) \quad \checkmark$$

$$\mathcal{M}(2^+000) = -\frac{1}{24\sqrt{6}} st(s+t).$$

$\Rightarrow$ massless limit and massive BCJ do not commute.

Double Copy at 5-Point

Again,

Feynman rules $\longrightarrow$ Partial amplitude $\longrightarrow$ BCJ numerators

- Unwieldy answer for numerators
- Study singularity structure **numerically**

Pick a set of independent $p_i \cdot p_j$, $\epsilon_i \cdot p_j$ and $\epsilon_i \cdot \epsilon_j$



Assign numerical values to all except one, say s_{12}



Study behaviour of amplitude around $s_{12} = -m^2$

- **Factorizes correctly on physical poles**

Note: Using the KLT form, we can show factorization on physical poles for **n -point amplitudes!**

Spurious Poles

BUT we have **spurious poles** in the denominator:

$$\alpha_1 s_{12}^4 + \alpha_2 s_{12}^3 + \alpha_3 s_{12}^2 + \alpha_4 s_{12} + \alpha_5 ,$$

where α_i are functions of the mass and other Mandelstam variables.

In special kinematic configurations, the exact locations of the spurious poles can be found and the amplitude evaluated on such a non-physical pole gives a **nonzero residue** $\Rightarrow$ (massive YM)² $\neq$ massive gravity.

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Where did these come from?

$$\mathcal{A}[\alpha] = M \cdot n$$

where M is a matrix of propagators i.e. contains **no spurious singularities**.

When do Spurious Poles Arise?

We find numerators by

$$n = M^{-1} \cdot \mathcal{A}[\alpha] = \frac{1}{\det M} (\text{matrix of cofactors}) \cdot \mathcal{A}[\alpha]$$

where the only source of spurious poles can be the **numerator of $\det M$** .

At 5-point, the numerator is proportional to

$$\begin{aligned} m^8 & \left(\binom{4}{s_{12}} + (2s_{13} + 2s_{14} + 2s_{23} + 2s_{24} - 6m^2) \binom{3}{s_{12}} + (s_{13}^2 + 2s_{13}s_{14} + 2s_{13}s_{23} + 4s_{13}s_{24} + s_{14}^2 + 4s_{14}s_{23} + 2s_{14}s_{24} + s_{23}^2 \right. \\ & + 2s_{23}s_{24} + s_{24}^2 - 6m^2s_{13} - 6m^2s_{14} - 6m^2s_{23} - 6m^2s_{24} + m^4) \binom{2}{s_{12}} + (2s_{13}^2s_{24} + 2s_{13}s_{14}s_{23} + 2s_{13}s_{14}s_{24} + 2s_{13}s_{23}s_{24} \\ & + 2s_{13}s_{24}^2 + 2s_{14}^2s_{23} + 2s_{14}s_{23}^2 + 2s_{14}s_{23}s_{24} + 4m^2s_{13}s_{14} - 4m^2s_{13}s_{23} - 6m^2s_{13}s_{24} - 6m^2s_{14}s_{23} - 4m^2s_{14}s_{24} \\ & + 4m^2s_{23}s_{24} - 8m^4s_{13} - 8m^4s_{14} - 8m^4s_{23} - 8m^4s_{24} + 24m^6) \binom{1}{s_{12}} + s_{13}^2s_{24}^2 - 2s_{13}s_{14}s_{23}s_{24} + s_{14}^2s_{23}^2 + 4m^2s_{13}^2s_{14} \\ & + 4m^2s_{13}s_{24}^2 + 4m^2s_{13}s_{14}s_{23} + 4m^2s_{13}s_{14}s_{24} + 4m^2s_{13}s_{23}s_{24} + 4m^2s_{14}s_{23}s_{24} + 4m^2s_{23}^2s_{24} + 4m^2s_{23}s_{24}^2 - 8m^4s_{13}^2 \\ & - 20m^4s_{13}s_{14} - 8m^4s_{13}s_{23} - 8m^4s_{13}s_{24} - 8m^4s_{14}^2 - 8m^4s_{14}s_{23} - 8m^4s_{14}s_{24} - 8m^4s_{23}^2 - 20m^4s_{23}s_{24} - 8m^4s_{24}^2 \\ & \left. + 24m^6s_{13} + 24m^6s_{14} + 24m^6s_{23} + 24m^6s_{24} - 20m^8 \right). \end{aligned}$$

Canceling the Spurious Poles

These spurious singularities **could** be canceled but they were **not** in $(\text{massive YM})^2 = (\text{massive gravity})$.

In the massless case,

BCJ relations $\Rightarrow$ Local amplitudes

In the massive case,

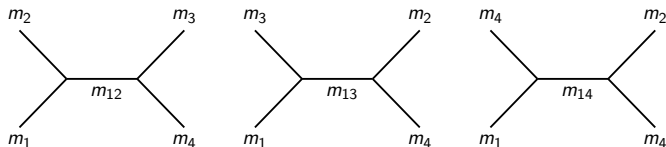
No BCJ relations $\Rightarrow$ Amplitudes with spurious poles

i.e. **extra condition** needs to be satisfied for locality

How else can we get rid of spurious poles? By considering a more general spectrum.

Spectral Condition

M has rank $(n - 3)!$ i.e. the following spectral condition holds:



$$m_{12}^2 + m_{13}^2 + m_{14}^2 = m_1^2 + m_2^2 + m_3^2 + m_4^2.$$

$\Rightarrow$ clearly **not** satisfied by mYM or **any other single mass spectrum**.

Satisfied by KK towers, Compton amplitudes, Bhabha scattering.

[Johansson, Ochirov, Chiodaroli, Gunaydin, Roiban, Guevara, Bautista...]

	Spectral Condition	Spurious Pole Cancellation
Local	✓	✓
Commutates with massless limit	✓	✗
BCJ Relations	✓	✗

Up Next:

- **Landscape exploration:** What theories satisfy the spectral condition/cancel the spurious singularities?
- **3 Dimensions:** Spurious pole-polynomial proportional to 3d Gram determinant... Can massive BCJ work in 3d?
- **Higher derivative corrections:** Can we add operators/dofs to massive YM to get a healthy double copy?
- **Multiple masses:** Generalize to case of multiple masses exchanged on each channel

Questions?