

Are Galileon Theories Supersymmetrizable?

Shruti Paranjape

University of Michigan, Ann Arbor
Leinweber Center for Theoretical Physics

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by H. Elvang, M. Hadjiantonis, C. R. T. Jones, SP

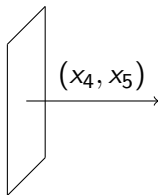
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Outline

- Goal: Classify and understand properties of low-energy EFTs arising from spontaneous symmetry breaking
- Method: Use on-shell tree-level amplitudes and their soft behaviour

Brane Teaser

- A 3-brane in a 6-d space
- Transverse directions x_4, x_5 , longitudinal directions $x^\mu = (x_0, x_1, x_2, x_3)$
- Embedding of the brane is described by two real scalar fields
 $\Rightarrow$ one complex scalar field ϕ



Spontaneously Broken Symmetries of ϕ

6-d Poincaré group is spontaneously broken $\Rightarrow$ **two Goldstone bosons $\phi, \bar{\phi}$**

- Translations P_4 and P_5 , e.g. $\phi \rightarrow \phi + a \partial_{x_4} \phi$
 $\Rightarrow$ Non-linearly realized as a shift symmetry $\phi \rightarrow \phi + c$
- Lorentz rotations $M_{\mu 4}$ and $M_{\mu 5}$
 $\Rightarrow$ Non-linearly realized as a spacetime-dependent shift symmetry
 $\phi \rightarrow \phi + c + v^\mu x_\mu$

Not obvious from the Lagrangian.

At leading order, only the brane tension term contributes

$$\mathcal{L}_{DBI} = \sqrt{-\det h_{\mu\nu}} = \sqrt{\det (\eta_{\mu\nu} - \partial_\mu \phi \partial_\nu \bar{\phi})}$$

where $h_{\mu\nu}$ is the induced metric on the brane.

DBI and Galileon

Schematically,

$$\mathcal{L}_{DBI} = \int d^4x \left(\frac{1}{2} \partial^2 \phi \bar{\phi} + g_4 \partial^4 \phi^2 \bar{\phi}^2 + g_6 \partial^6 \phi^3 \bar{\phi}^3 + \dots \right)$$

- ▷ Shift symmetry is 'trivial'
- ▷ Space-dependent shift symmetry is 'non-trivial'

Galileon theories

- ▷ Sub-leading terms to brane action
- ▷ Second order EOM
- ▷ Decoupling limit

$$\mathcal{L}_3 = \partial^4 \phi^2 \bar{\phi} + \text{h.c.}$$

$$\mathcal{L}_4 = \partial^6 \phi^2 \bar{\phi}^2$$

$$\mathcal{L}_5 = \partial^8 \phi^3 \bar{\phi}^2 + \text{h.c.}$$

Soft Behaviour

Symmetry

$$\phi \rightarrow \phi + c$$

$$\phi \rightarrow \phi + c + v^\mu x_\mu$$

Soft behaviour

$$\mathcal{A}_n(p_\phi) \sim p_\phi \text{ as } p_\phi \rightarrow 0.$$

$$\mathcal{A}_n(p_\phi) \sim p_\phi^2 \text{ as } p_\phi \rightarrow 0.$$

SSB of supersymmetry $\Rightarrow$ Goldstino ψ

Non-linear realization of broken SUSY is a shift $\psi \rightarrow \psi + \eta$, i.e.

$$\mathcal{A}_n(p_\psi) \sim p_\psi \quad \text{as} \quad p_\psi \rightarrow 0$$

Aim: Soft behaviour of amplitudes $\rightarrow$ Existence of SUSY Galileons

Supersymmetry

$\mathcal{N} = 1$ DBI has

- Soft behaviour p_ϕ^2 for $\phi, \bar{\phi}$
- Soft behaviour p_ψ for partner Goldstino
 $\Rightarrow$ Must break $\mathcal{N} = 2$ SUSY to $\mathcal{N} = 1$

What about the Galileon?

- ▷ Start with $\mathcal{L}_{\text{Gal}} = g_3 \mathcal{L}_3 + g_4 \mathcal{L}_4 + g_5 \mathcal{L}_5$
- ▷ Field redefinition $\phi \rightarrow \phi + a(\partial\phi)^2$
- ▷ Choose a so that $\tilde{g}_3 = 0$, i.e. $\mathcal{L}_{\text{Gal}} = \tilde{g}_4 \mathcal{L}_4 + \tilde{g}_5 \mathcal{L}_5$
- ▷ $\mathcal{N} = 1$ SUSY $\mathcal{L}_4$ in the next talk!

We answer the question of supersymmetrization of $\mathcal{L}_3$ via $\mathcal{L}_4$ and $\mathcal{L}_5$.

Supersymmetrizing the Quintic Galileon $\mathcal{L}_5$

- Find all structures for $\mathcal{A}_5(\phi_1, \bar{\phi}_2, \phi_3, \bar{\phi}_4, \phi_5)$
 - ▷ Lorentz invariance $\Rightarrow$ function of spinor brackets
 - ▷ Locality $\Rightarrow$ polynomial
 - ▷ Little group $\Rightarrow$ does not scale
 - ▷ Fermi-Bose symmetry $\Rightarrow$ symmetric under $(1 \leftrightarrow 3), (2 \leftrightarrow 4), (3 \leftrightarrow 5)$
 - ▷ Mass dimension $\Rightarrow [\mathcal{A}_5] = -9$
- Impose soft behaviour p_ϕ^2

$$\mathcal{A}_5(\phi_1, \bar{\phi}_2, \phi_3, \bar{\phi}_4, \phi_5) = \tilde{g}_5 [s_{13}s_{25} (s_{15}s_{23} + s_{12}s_{35} - s_{13}s_{25}) \\ + (1 \leftrightarrow 5) + (3 \leftrightarrow 5)] + (2 \leftrightarrow 4)$$

Supersymmetrizing the Quintic Galileon $\mathcal{L}_5$

Impose the SUSY Ward identities

$$\mathcal{A}_5(\phi_1, \bar{\phi}_2, \phi_3, \bar{\psi}_4, \psi_5) = \frac{[25]}{[24]} \mathcal{A}_5(\phi_1, \bar{\phi}_2, \phi_3, \bar{\phi}_4, \phi_5)$$

Has a pole as $[24] \rightarrow 0! \Rightarrow$ **not local**

There exists **no** $\mathcal{N} = 1$ SUSY $\mathcal{L}_5$ with p_ϕ^2 soft behaviour
 $\Rightarrow$ there is **no** supersymmetrization of $g_3 \mathcal{L}_3$

A SUSY quintic Galileon

- What about a theory having p_ϕ^2 behaviour for the real part of the complex scalar ϕ , but p_ϕ behaviour for the imaginary part?
- We find a consistent solution.
- Does such a theory make sense?
- p_ϕ^2 scalar is a Goldstone boson from 5d Poincaré group breaking
- p_ϕ scalar is an R-axion

Summary

- ▷ Galileons arise as terms subleading to DBI in brane actions
- ▷ We address the question of possible supersymmetrizations of Galileon theories via on-shell amplitudes
- ▷ We make definite statements of non-supersymmetrizability of $\mathcal{L}_3$ and $\mathcal{L}_5$
- ▷ We find evidence that supports the existence of a SUSY $\mathcal{L}_5$ with an R-axion

Thank you!