Supersymmetrizing Massive Gravity On-Shell

Shruti Paranjape

University of California, Davis
Center for Quantum Mathematics and Physics

ICTP Workshop on Scattering Amplitudes and Cosmology

Based on arXiv:2205.12982 with L Engelbrecht and C R T Jones

Why Modify Gravity?

- Cosmological constant: Can modified gravity explain acceleration?
- Hard to write down consistent modifications. Do they exist?
  E.g. Massive gravity, Galileons, bigravity, $f(R)$ gravity, etc.
- General relativity at long distances: Precision cosmology tests?
- "Islands" prescription might require a better understanding of massive gravity.

[Geng, Karch]

Let us modify gravity in the IR by giving the graviton a small constant mass: massive gravity.

[Hinterbichler][de Rham]

Lagrangians

$$S_{\mathrm{dRGT}}[h] = \frac{M_{\mathrm{P}}^2}{2}\int d^4x\sqrt{-g}\left(R[g] + m^2\left[S_2(\mathcal{K}) + \alpha_3 S_3(\mathcal{K}) + \alpha_4 S_4(\mathcal{K})\right]\right),$$

where

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_{\mathrm{P}}}h_{\mu\nu}\,,$$

$$S_n(\mathcal{K}) = n!\,\mathcal{K}^{[\mu_1}{}_{\mu_1}\mathcal{K}^{\mu_2}{}_{\mu_2}\cdots\mathcal{K}^{\mu_n]}{}_{\mu_n}\,,$$

$$\mathcal{K}^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - \sqrt{\delta^{\mu}{}_{\nu} - h^{\mu}{}_{\nu}} = -\sum_{n=1}^{\infty}\frac{(2n)!}{(1-2n)(n!)^2 4^n}(h^n)^{\mu}{}_{\nu}.$$

There are three parameters

- m: Mass of graviton
- α_3: Cubic dRGT coupling
- α_4: Quartic dRGT coupling

[de Rham, Gabadadze, Tolley]

Scales

- Large distances:

$$r \gtrsim r_C\,, \quad r_C = \frac{1}{m} \;\Rightarrow\; \text{Yukawa supression}$$

- Intermediate distances:

$$r_C \gtrsim r \gtrsim r_V\,, \quad r_V = \left(\frac{M}{M_P^2 m^2}\right)^{\frac{1}{3}} \;\Rightarrow\; \text{Modified gravity, Galileons}$$

- Short distances:

$$r_V \gtrsim r \gtrsim r_Q\,, \quad r_Q = \left(\frac{1}{M_P m^2}\right)^{\frac{1}{3}} \;\Rightarrow\; \text{Unmodified gravity}$$

- Very short distances:

$$r_Q \gtrsim r \;\Rightarrow\; ?$$

Is this Sensible?

Problems	Solutions
Diffeomorphism invariance broken	Restored in decoupling limit
BD ghosts Wrong dof count	dRGT potential 2-Parameter Lagrangian
vDVZ discontinuity Change in dof when $m \to 0$	Vainshtein mechanism GR within $r_V = \left(\frac{M}{M_P^2 m^2}\right)^{\frac{1}{3}}$
Closed/flat/open FLRW metrics Existence of solutions	Instabilities of such metrics Non-existence of solutions

Can Supersymmetry Help?

Two outstanding problems

- ▷ At loop-level, there is detuning of the dRGT parameters.

  [de Rham, Heisenberg, Ribeiro]

- ▷ UV completion of massive gravity models is unknown.

Possible solution: Supersymmetry

- ◦ Supersymmetry protects couplings/interactions from detuning.
- ◦ Most UV complete models involve some amount of supersymmetry.
- ◦ Might help search for hidden symmetries.

Massive Spinor-Helicity

Massive spinor helicity variables carry $SU(2)$ little group indices,

$$p_{i\mu}\sigma^{\mu}_{\alpha\dot{\alpha}} = p_{i\alpha\dot{\alpha}} = |i_I]_{\alpha}\langle i^I|_{\dot{\alpha}}$$

where the spinors satisfy

$$p_i|i^I] = -m|i^I\rangle\,, \qquad\qquad p_i|i^I\rangle = -m|i^I]$$

A spin s field is a symmetric $2s$-tensor in $SU(2)$ i.e.,

$$\phi,\ \lambda^I,\ \gamma^{IJ},\ \psi^{IJK},\ h^{IJKL}$$

Taking the high energy limit of h^{IJKL} will give us a tensor, vector and scalar mode corresponding to the 2, 1 and 0 spin modes.

[Arkani-Hamed, Huang, Huang]

On-Shell Supersymmetry

A representation of the SUSY algebra

$$\{Q_\alpha, Q^\dagger_{\dot\alpha}\} = -2\sigma^\mu_{\alpha\dot\alpha} P_\mu$$

can be given in terms of the massive spinor helicity variables:

$$Q^\dagger = -\sqrt{2}\sum_i |i^I\rangle \eta_{i,I}\,, \qquad Q = \sqrt{2}\sum_i |i_I]\frac{\partial}{\partial \eta_{i,I}}$$

which allow us to package supermultiplets into superfields based on different Clifford vacua

$$Q^\dagger_{\dot\alpha}|\Omega\rangle = 0\,.$$

For $\mathcal{N}$ supersymmetries, Q also carries an R-symmetry index, e.g.

$$Q^a_\alpha|\Omega\rangle = |\tilde\Omega^a\rangle\,.$$

[Herderschee, Koren, Trott]

Building Superfields

$\mathcal{N}=1$ massive graviton:

$$\Psi^{IJK}=\psi^{IJK}+\frac{1}{4\sqrt{3}}\eta^{(I}\gamma^{JK)}+\eta_L h^{IJKL}+\frac{1}{2}\eta_L\eta^L\tilde{\psi}^{IJK}.$$

$\mathcal{N}=4$ massive graviton:

$$\begin{aligned}\Phi=\;&\phi+\eta_{aI}\lambda^{aI}+\frac{1}{2}\eta_{aI}\eta_b^I\phi^{ab}+\frac{1}{2\sqrt{2}}\epsilon^{abcd}\eta_{aI}\eta_{bJ}\gamma^{IJ}_{cd}+\frac{1}{3\sqrt{2}}\eta_{aI}\eta_b^{(I}\eta_{cJ}\lambda^{abcJ)}\\&+\frac{1}{6}\epsilon^{abcd}\eta_{aI}\eta_{bJ}\eta_{cK}\psi_d^{IJK}+\frac{1}{12}\eta_{aI}\eta_b^{(I}\eta_{cJ}\eta_d^{J)}\phi^{abcd}+\frac{1}{8\sqrt{6}}\eta_{aI}\eta_b^{(I}\eta_{cJ}\eta_{dK}\gamma^{abcdJK)}\\&+\frac{1}{24}\epsilon^{abcd}\eta_{aI}\eta_{bJ}\eta_{cK}\eta_{dL}h^{IJKL}+\frac{1}{30}\epsilon^{abcd}\eta_{aI}\eta_{bJ}\eta_{cK}\eta_{dL}\eta_e^L\psi^{eIJK}\\&+\frac{1}{24\sqrt{3}}\eta_{aI}\eta_b^{(I}\eta_{cJ}\eta_d^J\eta_{eK}\lambda^{K)abcde}+\frac{1}{144}\epsilon^{abcd}\epsilon^{efgh}\eta_{aI}\eta_{bJ}\eta_{cK}\eta_e^I\eta_f^J\eta_g^K\phi_{dh}\\&+\frac{1}{80}\epsilon^{abcd}\eta_{aI}\eta_{bJ}\eta_{cK}\eta_{dL}\eta_e^I\eta_f^J\gamma^{efKL}+\frac{1}{360}\epsilon^{abcd}\epsilon^{efgh}\eta_{aI}\eta_{bJ}\eta_{cK}\eta_{dL}\eta_e^I\eta_f^J\eta_g^K\lambda_h^L\\&+\frac{1}{2880}\epsilon^{abcd}\epsilon^{efgh}\eta_{aI}\eta_{bJ}\eta_{cK}\eta_{dL}\eta_e^I\eta_f^J\eta_g^K\eta_h^L\tilde{\phi}.\end{aligned}$$

R-symmetry Representations

Field	Spin	$U(1)_R$	Dim.
ψ^{IJK}	$\frac{3}{2}$	1	**1**
γ^{IJ}	1	0	**1**
h^{IJKL}	2	0	**1**
$\tilde{\psi}^{IJK}$	$\frac{3}{2}$	-1	**1**

Field	Spin	$U(1)_R$	$SU(2)_R$	Dim.
γ^{IJ}	1	2	$\bullet$	**1**
λ^{aI}	$\frac{1}{2}$	1	$\square$	**2**
ψ^{aIJK}	$\frac{3}{2}$	1	$\square$	**2**
h^{IJKL}	2	0	$\bullet$	**1**
γ^{abIJ}	1	0	$\square\square$	**3**
V^{IJ}	1	0	$\bullet$	**1**
ϕ	0	0	$\bullet$	**1**
$\tilde{\lambda}^{aI}$	$\frac{1}{2}$	-1	$\square$	**2**
$\tilde{\psi}^{aIJK}$	$\frac{3}{2}$	-1	$\square$	**2**
$\tilde{\gamma}^{IJ}$	1	-2	$\bullet$	**1**

R-symmetry Representations

Field	Spin	$U(1)_R$	$SU(3)_R$	Dim.
λ^I	$\frac{1}{2}$	3	$\bullet$	**1**
ϕ^a	0	2	$\ydiagram{1}$	**3**
γ^{aIJ}	1	2	$\ydiagram{1}$	**3**
λ^{abI}	$\frac{1}{2}$	1	$\ydiagram{2}$	$\overline{\mathbf{6}}$
λ^I_a	$\frac{1}{2}$	1	$\ydiagram{1,1}$	$\overline{\mathbf{3}}$
ψ^{IJK}_a	$\frac{3}{2}$	1	$\ydiagram{1,1}$	$\overline{\mathbf{3}}$
ϕ^{abc}	0	0	$\ydiagram{2,1}$	**8**
γ^{IJ}	1	0	$\bullet$	**1**
γ^{abcIJ}	1	0	$\ydiagram{2,1}$	**8**
h^{IJKL}	2	0	$\bullet$	**1**
λ^I_{ab}	$\frac{1}{2}$	-1	$\ydiagram{2,2}$	**6**
λ^{aI}	$\frac{1}{2}$	-1	$\ydiagram{1}$	**3**
ψ^{aIJK}	$\frac{3}{2}$	-1	$\ydiagram{1}$	**3**
γ^{IJ}_a	1	-2	$\ydiagram{1,1}$	$\overline{\mathbf{3}}$
ϕ_a	0	-2	$\ydiagram{1,1}$	$\overline{\mathbf{3}}$
$\tilde{\lambda}^I$	$\frac{1}{2}$	-3	$\bullet$	**1**

Field	Spin	$U(1)_R$	$SU(4)_R$	Dim.
ϕ	0	4	$\bullet$	**1**
λ^{aI}	$\frac{1}{2}$	3	$\ydiagram{1}$	**4**
ϕ^{ab}	0	2	$\ydiagram{2}$	**10**
γ^{IJ}_{ab}	1	2	$\ydiagram{1,1}$	**6**
λ^{abcI}	$\frac{1}{2}$	1	$\ydiagram{2,1}$	$\overline{\mathbf{20}}$
ψ^{IJK}_a	$\frac{3}{2}$	1	$\ydiagram{1,1,1}$	$\overline{\mathbf{4}}$
ϕ^{abcd}	0	0	$\ydiagram{2,2}$	$\mathbf{20'}$
γ^{abcdIJ}	1	0	$\ydiagram{2,1,1}$	**15**
h^{IJKL}	2	0	$\bullet$	**1**
ψ^{aIJK}	$\frac{3}{2}$	-1	$\ydiagram{1}$	**4**
λ^{abcdeI}	$\frac{1}{2}$	-1	$\ydiagram{2,2,1}$	**20**
ϕ_{ab}	0	-2	$\ydiagram{2,2,2}$	$\overline{\mathbf{10}}$
γ^{abIJ}	1	-2	$\ydiagram{1,1}$	**6**
λ^I_a	$\frac{1}{2}$	-3	$\ydiagram{1,1,1}$	$\overline{\mathbf{4}}$
$\tilde{\phi}$	0	-4	$\bullet$	**1**

Superamplitudes

Amplitudes must satisfy the supersymmetry Ward identities corresponding to Q and $Q^\dagger$. Since $Q^\dagger$ is multiplicative,

$$\delta^{(2)}\left(Q^\dagger\right) = \sum_{i<j} \langle i^I j^J \rangle \eta_{i,I}\eta_{j,J} + \sum_i \frac{m_i^2}{2}\eta_{i,I}\eta_i^I \, .$$

In addition, the superamplitude can depend on a polynomial of Grassmann variables that satisfies

$$Q \cdot \text{ polynomial in } \eta = 0 \Rightarrow \mathcal{A}_n \propto \delta^{(2)}\left(Q^\dagger\right) \times \text{ polynomial in } \eta_{ij}^I$$

where $\eta_{ij}^I = \eta_i^I - \frac{1}{m}[i^I j_J]\eta_j^J$.

We are interested in gravitons which are R-singlets so

$$\mathcal{A}_3(\Phi^{\{I_{\text{ext}}\}}) \supset \delta^{(2\mathcal{N})}\left(Q^\dagger\right) F^{\{I_{\text{ext}}\}J_1\cdots J_{\mathcal{N}}}\eta_{12,J_1}\cdots\eta_{12,J_{\mathcal{N}}}$$

Methodology

1. Write down an ansatz for F i.e. all combinations of kinematical quantities that have the correct little group structure
2. Impose *R*-symmetry constraints e.g. for $\mathcal{N}=4$

$$\mathcal{A}_3\left(\Phi\Phi\Phi\right) = \prod_{a=1}^{4} \delta^{(2)}\left(Q^{a\dagger}\right) F^{I_1 J_1 K_1 L_1} \epsilon_{abcd} \prod_{\substack{e\in\{a,b,c,d\} \\ M_1\in\{I_1 J_1 K_1 L_1\}}} \eta^{e}_{12,M_1}$$

$\Rightarrow F^{I_1 J_1 K_1 L_1}$ is symmetric in I_1, J_1, K_1, L_1

3. Impose exchange symmetry constraints e.g.

$\mathcal{A}_3^{\mathcal{N}=1}$ is anti-symmetric under the exchange of $\Psi^{I_1 J_1 K_1}$ and $\Psi^{I_2 J_2 K_2}$

Possible 3-Graviton Interactions

$$\mathcal{L}_1 = \frac{m^2}{3M_\mathrm{P}} h_\mu{}^\nu h_\nu{}^\lambda h_\lambda{}^\mu \quad \text{(dRGT potential)} \quad \rightarrow \quad 2\mathcal{B}_1\,,$$

$$\mathcal{L}_2 = \frac{M_\mathrm{P}^2}{2}\sqrt{-g}R|_{(3)} \quad \text{(Einstein-Hilbert)} \quad \rightarrow \quad -6\mathcal{B}_1 + 2\mathcal{B}_2\,,$$

$$\mathcal{L}_3 = \frac{1}{M_\mathrm{P}}\epsilon^{\mu\nu\lambda\rho}\epsilon^{\alpha\beta\gamma\delta}\partial_\mu\partial_\alpha h_{\nu\beta}h_{\lambda\gamma}h_{\rho\delta} \quad \rightarrow \quad 12\mathcal{B}_1 - 2\mathcal{B}_2 - 2\mathcal{B}_3\,,$$

$$\mathcal{L}_4 = \frac{M_\mathrm{P}^2}{m^4}\sqrt{-g}R_{\mu\nu}{}^{\alpha\beta}R_{\alpha\beta}{}^{\lambda\rho}R_{\lambda\rho}{}^{\mu\nu}|_{(3)} \quad \rightarrow \quad 24\mathcal{B}_1 - 12\mathcal{B}_3 + 48\mathcal{B}_4\,,$$

$$\mathcal{L}_5 = \frac{1}{M_\mathrm{P}}\epsilon^{\mu\nu\lambda\rho}\partial_\mu h_{\nu\alpha}\partial_\lambda h_{\rho\beta}h^{\alpha\beta} \quad \rightarrow \quad 2\mathcal{B}_5\,,$$

$$\mathcal{L}_6 = \frac{M_\mathrm{P}^2}{m^4}\sqrt{-g}\widetilde{R}_{\mu\nu}{}^{\alpha\beta}R_{\alpha\beta}{}^{\lambda\rho}R_{\lambda\rho}{}^{\mu\nu}|_{(3)} \quad \rightarrow \quad -8\mathcal{B}_5 - 32\mathcal{B}_6\,,$$

where we have chosen a basis of monomials in ε^μ and p^μ as our basis $\mathcal{B}_i$.

$\mathcal{N} = 1$

The amplitude is $\mathcal{A}_3(\Psi^{I_iJ_iK_i}) = \delta^{(2)}\left(Q^\dagger\right) F^{\{I_iJ_iK_i\}L}\eta_{12,L}$ where

$$
\begin{aligned}
F^L_{\mathcal{N}=1} = \beta_1 \Big(& \{\mathbf{33}\}(\{1^L\mathbf{3}\} - 2\{\mathbf{3}1^L\})\{\mathbf{12}\}^3 + 3\{\mathbf{13}\}^2\{\mathbf{23}\}\{1^L\mathbf{2}\}\{\mathbf{12}\} + \{\mathbf{11}\}(\{\mathbf{12}\}((3\{\mathbf{3}1^L\} \\
& - \{1^L\mathbf{3}\})\{\mathbf{23}\}^2 - 2(\{\mathbf{2}1^L\}\{\mathbf{33}\} + \{\mathbf{32}\}\{\mathbf{3}1^L\})\{\mathbf{23}\} + \{\mathbf{22}\}\{\mathbf{33}\}(2\{\mathbf{3}1^L\} - \{1^L\mathbf{3}\})) \\
& + \{\mathbf{13}\}\{\mathbf{23}\}(\{\mathbf{23}\}(2\{\mathbf{2}1^L\} - \{1^L\mathbf{2}\}) - \{\mathbf{22}\}(\{\mathbf{3}1^L\} + \{1^L\mathbf{3}\})))\Big) \\
+ \beta_2 \Big(& \{\mathbf{33}\}(2\{\mathbf{3}1^L\} - \{1^L\mathbf{3}\})\{\mathbf{12}\}^3 + 3\{\mathbf{13}\}\{\mathbf{23}\}(\{1^L\mathbf{3}\} - 2\{\mathbf{3}1^L\})\{\mathbf{12}\}^2 \\
& + (3\{\mathbf{23}\}\{\mathbf{2}1^L\}\{\mathbf{13}\}^2 + \{\mathbf{11}\}((6\{\mathbf{3}1^L\} - 5\{1^L\mathbf{3}\})\{\mathbf{23}\}^2 - (\{\mathbf{2}1^L\}\{\mathbf{33}\} \\
& + \{\mathbf{32}\}\{\mathbf{3}1^L\})\{\mathbf{23}\} + \{\mathbf{22}\}\{\mathbf{33}\}(\{1^L\mathbf{3}\} - 2\{\mathbf{3}1^L\})))\{\mathbf{12}\} \\
& + \{\mathbf{11}\}\{\mathbf{13}\}\{\mathbf{23}\}(\{\mathbf{23}\}(\{1^L\mathbf{2}\} - 2\{\mathbf{2}1^L\}) + \{\mathbf{22}\}(\{\mathbf{3}1^L\} + \{1^L\mathbf{3}\}))\Big) \\
+ \beta_3 \Big(& (\{\mathbf{13}\}(-\{\mathbf{2}1^L\}\{\mathbf{33}\} + 2\{\mathbf{32}\}\{\mathbf{3}1^L\} - 2\{\mathbf{23}\}(\{\mathbf{3}1^L\} - \{1^L\mathbf{3}\})) + \{\mathbf{33}\}\{\mathbf{21}\}\{\mathbf{3}1^L\} \\
& - \{\mathbf{12}\}\{1^L\mathbf{3}\}))\{\mathbf{12}\}^2 + \{\mathbf{11}\}\{\mathbf{23}\}(-\{\mathbf{1}1^L\}\{\mathbf{22}\}\{\mathbf{33}\} + \{\mathbf{12}\}(\{\mathbf{2}1^L\}\{\mathbf{33}\} \\
& - 2\{\mathbf{32}\}\{\mathbf{3}1^L\} + 2\{\mathbf{23}\}(\{\mathbf{3}1^L\} - \{1^L\mathbf{3}\})) + \{\mathbf{13}\}\{\mathbf{22}\}\{1^L\mathbf{3}\})\Big) \\
+ \beta_4 \Big(& \{\mathbf{11}\}\{\mathbf{22}\}(-2\{\mathbf{1}1^L\}\{\mathbf{23}\}\{\mathbf{33}\} + \{\mathbf{21}\}\{\mathbf{3}1^L\} + \{\mathbf{12}\}(\{\mathbf{3}1^L\} - \{1^L\mathbf{3}\}))\{\mathbf{33}\} \\
& + \{\mathbf{13}\}(\{\mathbf{32}\}\{\mathbf{3}1^L\} + \{\mathbf{23}\}(\{1^L\mathbf{3}\} - \{\mathbf{3}1^L\})))\Big) .
\end{aligned}
$$

where $\{ij\} = [i|p_1 p_2|j]$.

$\mathcal{N} = 1$

The most general amplitude is

$$\mathcal{A}_3(hhh) = \sum_{i=1}^{6} b_i \mathcal{B}_i$$

and $\mathcal{N} = 1$ picks out

$$\begin{aligned}
b_1 =& -192\, m^{14} M_{\mathrm{P}} \left(\beta_1 + 2\beta_2 - \beta_3 - 2\beta_4\right) ,\\
b_2 =& -64\, m^{14} M_{\mathrm{P}} \left(\beta_1 + 2\beta_2 + 2\beta_3 + 2\beta_4\right) ,\\
b_3 =& 64\, m^{14} M_{\mathrm{P}} \left(\beta_1 + 2\beta_2 - 2\beta_4\right) ,\\
b_5 =& 96\, i\, m^{14} M_{\mathrm{P}} \beta_1 ,\\
b_4 =& b_6 = 0 .
\end{aligned}$$

Translating this into a statement about the local operator basis:

Arbitrary linear combinations of the operators $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3$ and $\mathcal{L}_5$ are supersymmetrizable.

Results at 3-Point

	$\mathcal{N}=1$	$\mathcal{N}=2$	$\mathcal{N}=3$	$\mathcal{N}=4$
Parameters in ansatz	24	19	8	1
Parameters after super-statistics	4	5	2	1
Parameters in $A_3(h,h,h)$	4	4	2	1
Supersymmetrizable amplitudes	$\mathcal{B}_1$,$\mathcal{B}_2$,$\mathcal{B}_3$,$\mathcal{B}_5$	$\mathcal{B}_1$,$\mathcal{B}_2$,$\mathcal{B}_3$,$\mathcal{B}_5$	$\mathcal{B}_1$,$\mathcal{B}_2$	$\mathcal{B}_2$
Supersymmetrizable operators	$\mathcal{L}_1$,$\mathcal{L}_2$,$\mathcal{L}_3$,$\mathcal{L}_5$	$\mathcal{L}_1$,$\mathcal{L}_2$,$\mathcal{L}_3$,$\mathcal{L}_5$	$\mathcal{L}_1$,$\mathcal{L}_2$	$3\mathcal{L}_1+\mathcal{L}_2$

- $\mathcal{L}_4$ and $\mathcal{L}_6$ i.e. Riemann^3 and its parity-violating analog are incompatible with any amount of supersymmetry.
- For $\mathcal{N}\geq 3$, only dRGT is allowed.
- Maximal $\mathcal{N}=4$ supersymmetry selects a particular relation between the dRGT parameters that is causal.

[Camanho, Lucena Gómez, Rahman][Bonifacio, Hinterbichler, Joyce, Rosen]

$\mathcal{N}=3$

The amplitude is $\mathcal{A}_3(\Lambda^{I_i}) = \delta^{(6)}\left(Q^\dagger\right) F^{\{I_i\}JKL}\eta^1_{12,J}\eta^2_{12,K}\eta^3_{12,L}$ where

$$
\begin{aligned}
F^{JKL}_{\mathcal{N}=3} =\beta_1\Big(& -2\{\mathbf{1}1^L\}\{\mathbf{23}\}\{1^J1^K\} + \{\mathbf{13}\}\{\mathbf{2}1^L\}\{1^J1^K\} + \{\mathbf{1}1^K\}\{\mathbf{23}\}\{1^J1^L\} \\
& - 3\{\mathbf{13}\}\{\mathbf{2}1^K\}\{1^J1^L\} + 2\{\mathbf{12}\}\{\mathbf{3}1^K\}\{1^J1^L\} + \{\mathbf{13}\}\{\mathbf{2}1^J\}\{1^K1^L\} \\
& + 2\{\mathbf{21}\}\{\mathbf{3}1^J\}\{1^K1^L\} + 2\{\mathbf{13}\}\{1^J\mathbf{2}\}\{1^K1^L\} - 2\{\mathbf{12}\}\{1^J\mathbf{3}\}\{1^K1^L\} \\
& - \{\mathbf{1}1^K\}\{\mathbf{23}\}\{1^L1^J\} + \{\mathbf{13}\}\{\mathbf{2}1^K\}\{1^L1^J\} - \{\mathbf{12}\}\{\mathbf{3}1^K\}\{1^L1^J\} \\
& + \{\mathbf{12}\}\{\mathbf{3}1^J\}\{1^L1^K\} + \{\mathbf{1}1^J\}(3\{\mathbf{2}1^K\}\{\mathbf{3}1^L\} - \{\mathbf{23}\}(4\{1^K1^L\} + \{1^L1^K\}))\Big) \\
+ \beta_2\Big(& -\{\mathbf{1}1^L\}\{\mathbf{23}\}\{1^J1^K\} + (-\{\mathbf{1}1^J\}\{\mathbf{23}\} + \{\mathbf{21}\}\{\mathbf{3}1^J\} + \{\mathbf{13}\}\{1^J\mathbf{2}\})\{1^K1^L\} \\
& + \{\mathbf{12}\}(\{\mathbf{3}1^L\}\{1^J1^K\} - \{1^J\mathbf{3}\}\{1^K1^L\})\Big)\,.
\end{aligned}
$$

which gives a 3-graviton amplitude

$$
\begin{aligned}
b_1 &= -\,192\, m^{10}\, M_{\mathrm{P}}\beta_1\,, \\
b_2 &= 128\, m^{10}\, M_{\mathrm{P}}(7\beta_1 + 2\beta_2)\,, \\
b_3 &= b_4 = b_5 = b_6 = 0\,.
\end{aligned}
$$

$\boxed{\text{Arbitrary linear combinations of the operators } \mathcal{L}_1 \text{ and } \mathcal{L}_2 \text{ are supersymmetrizable.}}$

$\mathcal{N} = 4$

The amplitude is $\mathcal{A}_3(\Phi) = \delta^{(8)}\left(Q^\dagger\right) F^{IJKL} \eta^1_{12,I} \eta^2_{12,J} \eta^3_{12,K} \eta^4_{12,L}$ where

$$F^{I_1 J_1 K_1 L_1}_{\mathcal{N}=4} = \beta_1 \left(\{1^{I_1} 1^{K_1}\}\{1^{L_1} 1^{J_1}\} + \{1^{J_1} 1^{I_1}\}\{1^{K_1} 1^{L_1}\} \right).$$

Projecting onto the cubic graviton amplitude,

$$\begin{aligned} b_2 =& 256\, m^8\, M_{\mathrm{P}} \beta_1\,, \\ b_1 =& b_3 = b_4 = b_5 = b_6 = 0\,. \end{aligned}$$

$\boxed{\text{Only the linear combination } 3\mathcal{L}_1 + \mathcal{L}_2 \text{ is supersymmetrizable.}}$

Decoupling Limit of Amplitudes

$$\mathcal{B}_1 \xrightarrow{\text{HE}} \begin{cases} (h^- v^- \phi): & -\frac{1}{M_{\text{P}} m} \frac{\langle 12\rangle^3 \langle 13\rangle}{\langle 23\rangle} \\ (h^+ v^+ \phi): & -\frac{1}{M_{\text{P}} m} \frac{[12]^3 [13]}{[23]} \end{cases}$$

$$\mathcal{B}_2 \xrightarrow{\text{HE}} \begin{cases} (h^- h^- h^+): & \frac{1}{M_{\text{P}}} \frac{\langle 12\rangle^6}{\langle 13\rangle^2 \langle 23\rangle^2} \\ (h^- v^- v^+): & -\frac{2}{M_{\text{P}}} \frac{\langle 12\rangle^4}{\langle 23\rangle^2} \\ (h^- \phi\phi): & \frac{3}{M_{\text{P}}} \frac{\langle 12\rangle^2 \langle 13\rangle^2}{\langle 23\rangle^2} \\ (h^+ \phi\phi): & \frac{3}{M_{\text{P}}} \frac{[12]^2 [13]^2}{[23]^2} \\ (h^+ v^+ v^-): & -\frac{2}{M_{\text{P}}} \frac{[12]^4}{[23]^2} \\ (h^+ h^+ h^-): & \frac{1}{M_{\text{P}}} \frac{[12]^6}{[13]^2 [23]^2} \end{cases}$$

Interestingly, constraints of massless supersymmetry rule out the same interactions as massive supersymmetry.

What are these scalars?

How to Identify the Galileon

Galileon is a massless scalar ϕ that arises in the decoupling limit of dRGT from the $s = 0$ mode of the massive spin-2 h^{IJKL}.

h^{IJKL} does not carry any R-charge $\Rightarrow \phi$ must be $\mathbf{1}_0$

Looking at R-symmetry of the massless multiplets, we can identify which combination is non-chiral and an R-singlet.

E.g. For $\mathcal{N} = 4$ massive graviton, the Galileon mode is uniquely identified with a linear combination of massless *vector* multiplets

$$\phi^{(\mathcal{N}=4 \text{ Galileon})} \propto \epsilon^{ijab}\phi_{ij,ab}.$$

What else is special about supersymmetric massive gravity?

Can it be a double copy?

Massive Spin-1 Superfields

The superfields now have color.

Field	Spin	$U(1)_R$	Dim.
λ^I	$\frac{1}{2}$	1	1
g^{IJ}	1	0	1
H	0	0	1
$\tilde{\lambda}^I$	$\frac{1}{2}$	-1	1

Field	Spin	$U(1)_R$	$SU(2)_R$	Dim.
ϕ	0	2	$\bullet$	**1**
ψ_a^I	$\frac{1}{2}$	1	$\square$	**2**
g^{IJ}	1	0	$\bullet$	**1**
H_{ab}	0	0	$\square\square$	**3**
$\tilde{\psi}_a^I$	$\frac{1}{2}$	-1	$\square$	**2**
$\tilde{\phi}$	0	-2	$\bullet$	**1**

Massive Spin-1 Interactions

Most general massive spin-1 amplitudes are given by

$$\hat{\mathcal{L}}_1 = \frac{f_{abc}}{m^2} F^{a\mu\rho} F_\rho^{b\nu} F^c_{\mu\nu}|_{(3)} \quad \rightarrow \quad \mathcal{C}_1$$

$$\hat{\mathcal{L}}_2 = F^a_{\mu\nu} F_a^{\mu\nu}|_{(3)} \quad \rightarrow \quad \mathcal{C}_2$$

$$\hat{\mathcal{L}}_3 = \frac{f_{abc}}{m^2} \epsilon_{\mu\nu\alpha\beta} F^{a\mu\rho} F_\rho^{b\nu} F^{c\alpha\beta}|_{(3)} \quad \rightarrow \quad \mathcal{C}_3.$$

$$\mathcal{C}_1 \xrightarrow{\text{HE}} \begin{cases} (g^- g^- g^-) : & \frac{1}{m^2} \langle 12\rangle\langle 13\rangle\langle 23\rangle \\ (g^+ g^+ g^+) : & \frac{1}{m^2} [12][13][23] \end{cases}$$

$$\mathcal{C}_2 \xrightarrow{\text{HE}} \begin{cases} (g^- g^- g^+) : & \frac{\langle 12\rangle^3}{\langle 13\rangle\langle 23\rangle} \\ (g^- \pi\pi) : & -\frac{\langle 12\rangle\langle 13\rangle}{\langle 23\rangle} \\ (g^+ \pi\pi) : & -\frac{[12][13]}{[23]} \\ (g^+ g^+ g^-) : & \frac{[12]^3}{[13][23]} \end{cases}$$

Double Copy of Massive Superfields

	Double Copy	Fields Generated	R-Symmetry
$\mathcal{N} = 0 + 0$	$g^{(IJ} \otimes g^{KL)}$ $\epsilon^{KL} g_K^{(I} \otimes g_L^{J)}$ $g^{IJ} \otimes g_{IJ}$	Graviton h^{IJKL} Massive 2-form B^{IJ} Dilaton D	$--$
$\mathcal{N} = 1 + 0$	$\Pi^{(I} \otimes g^{JK)}$ $\Pi_J \otimes g^{IJ}$	Graviton superfield Ψ^{IJK} Vector superfield V^I	$U(1)$
$\mathcal{N} = 2 + 0$	$\Theta \otimes g^{IJ}$	Graviton superfield Γ^{IJ}	$U(2)$
$\mathcal{N} = 1 + 1$	$\Pi^{(I} \otimes \Pi^{J)}$ $\epsilon_{IJ} \Pi^I \otimes \Pi^J$	Graviton superfield Γ^{IJ} Vector superfield W	$U(1) \times U(1)$
$\mathcal{N} = 2 + 1$	$\Theta \otimes \Pi^I$	Graviton superfield Λ^I	$U(2) \times U(1)$
$\mathcal{N} = 2 + 2$	$\Theta \otimes \Theta$	Graviton superfield Φ	$U(2) \times U(2)$

Massive Gravity Interactions as a Double Copy

Double copy	Result	Parameter Space
$\mathcal{N}=0\times\mathcal{N}=0$	$\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3$, $\mathcal{L}_4$, $\mathcal{L}_5$, $\mathcal{L}_6$	YM picks out causal value
$\mathcal{N}=1\times\mathcal{N}=0$	$\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_5$	Generically 4 parameter space
$\mathcal{N}=2\times\mathcal{N}=0$	$\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_5$	Generically 4 parameter space
$\mathcal{N}=1\times\mathcal{N}=1$	$3\mathcal{L}_1+\mathcal{L}_2$	Causal value
$\mathcal{N}=2\times\mathcal{N}=1$	$3\mathcal{L}_1+\mathcal{L}_2$	Causal value
$\mathcal{N}=2\times\mathcal{N}=2$	$3\mathcal{L}_1+\mathcal{L}_2$	Causal value

Comparison with $\mathcal{N} = 8$ SUGRA

- Both maximally supersymmetric graviton superfields can be constructed as a double copy
- Both generate the unique 3-point interaction compatible with the symmetries
- Both superfields contain the dilaton as a scalar component

- BUT the double copy of $\mathcal{N} = 8$ SUGRA has enhanced R-symmetry from $SU(4) \times SU(4)$ to $SU(8)$
- $\mathcal{N} = 8$ SUGRA also has non-linearly realized E_7 symmetry- is there any such symmetry from maximally supersymmetric massive gravity?

Constructing 4-Point Amplitudes

We can construct manifestly supersymmetric pole terms from lower-point amplitudes:

$$
\begin{aligned}
&-\delta^{(2)}(Q^\dagger_{1234})F^{\{I_1\}\{I_2\}\{I_P\}K_1}F^{\{I_3\}\{I_4\}L_3}_{\{I_P\}} \\
&\qquad \times \left([3_{L_3}|p_4 p_2|1^{L_1}]\epsilon_{K_1 L_1}\eta^2_{12} + \left([3_{L_3}1^{L_1}] + [3_{L_3}|p_4 p_3|1^{L_1}]\right)\eta_{12,K_1}\eta_{13,L_1}\right)
\end{aligned}
$$

- Contact terms are no longer finite.
- Must be expanded in a derivative expansion.
- One needs to go to 6-derivative order to understand dRGT.
- Multiple Grassmann polynomial structures.

Next Questions

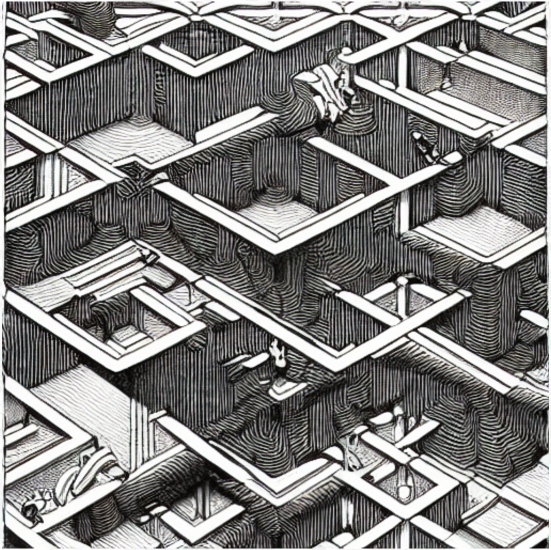
- What will 4-point amplitudes tell us? Is the dRGT potential protected by supersymmetry?
- Bigravity theories were shown to arise from certain string theories. Can these be supersymmetrized?

  [Lust, Markou, Mazloumi, Steiberger]
- The twisted heterotic string amplitudes naturally involve supersymmetric spin-2 particles. What does a similar analysis have to say about these amplitudes?

  [Guillen, Johansson, Jusinskas, Schlotterer]
- Is there a UV complete theory that gives rise to the Galileon in the low-energy limit?

Summary

- On-shell methods are effective in constructing supersymmetrizations of massive gravity
- Increasing amounts of supersymmetry give place stricter constraints on possible 3-point interactions and their corresponding Lagrangians.
- Galileon modes can be identified by looking for uncharged modes in the decoupling limit.
- $\mathcal{N} = 4$ picks out an amplitude that can be double-copied and parameter values consistent with the lack of superluminal propagation.



Thank You