Double Copy, Pions and EFTs

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Discussion at "What is Particle Theory?"

Last Week

We heard about the double copy.

see talk by Henrik Johansson

This is a map between gauge and gravity theories:

- Open and closed string amplitudes
- Gluon and graviton amplitudes
- Classical solutions of Yang-Mills and gravity solutions
- SYM and supergravities with different amounts of matter and SUSY
- Other colored and uncolored theories

There are *many* applications of the double copy to gravity and supergravity.

see reviews [1909.01358] [2203.13013] [2204.06547]

Today

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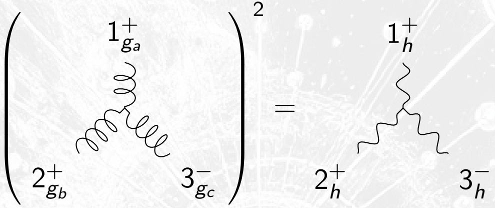
Outline:

- Review the basics
- Extend color-kinematics and the double copy to EFTs
- Extend them to theories with mass
- Discuss hidden zeros as a consequence of the double copy
- Further scope

Double Copy: Color-kinematic Recap

Recall that at the level of 3-point amplitudes, the double copy is very simple:



Yet beyond 3-point, we needed color-kinematic duality, which becomes a statement of these non-gauge invariant constituents of the amplitude.

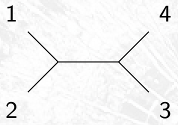
Is there a gauge-invariant formulation of the double copy?

Double Copy: Color-kinematic Recap

Consider a YM amplitude

$$\mathcal{A}_n^{\text{YM}} = \sum_i \frac{c_i n_i}{P_i}$$

$\mathcal{A}_4^{\text{YM}} =$



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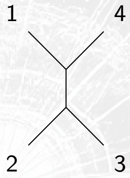

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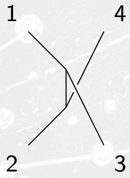

$$\frac{f^{12c} f^{c34} n_s}{s} \qquad \frac{f^{14c} f^{c32} n_u}{u} \qquad \frac{f^{13c} f^{c42} n_t}{t}$$

Double Copy: Color-kinematic Recap

Find a representation such that:

$$\text{Jacobi identity } c_i + c_j + c_k = 0$$
$$\Downarrow$$
$$\text{kinematic Jacobi identity } n_i + n_j + n_k = 0$$

Obtain a gravity amplitude as the double copy:

$$\mathcal{A}_n^{\text{GR}} = \sum_i \frac{n_i n_i}{P_i}$$

$$\text{e.g.} \quad \mathcal{A}_4^{\text{GR}} = \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

Generalizes to loop-level!

[Bern, Carrasco, Johansson]

Double Copy: Using Partial amplitudes

Trace decomposition:

$$\mathcal{A}_n(1^{a_1}\cdots n^{a_n}) = \sum_{\sigma} \mathcal{A}_n[1\sigma(2\cdots n)]\mathrm{Tr}\left(T^{a_1}T^{\sigma(a_2}\cdots T^{a_n)}\right)$$

There are $(n-1)!$ partial amplitudes, $(n-2)!$ if the Lagrangian only has f^{abc} and it further reduces to $(n-3)!$ due to the statement of color-kinematics duality. This becomes the BCJ relations:

$$A_4[1234] = \frac{t}{s}A_4[1324]$$

Put color-ordered YM amplitudes together with kinematic factors (the KLT kernel) to get gravity amplitudes:

$$\mathcal{A}_4^{\mathrm{GR}}(1234) = \frac{us}{t}\mathcal{A}_4^{\mathrm{YM}}[1234]^2$$

[Kawai, Lewellen, Tye][Del Duca, Dixon, Maltoni]

How is this related to pions?

Scalar Double Copies

	ϕ^3	χPT	YM
ϕ^3	ϕ^3	χPT	YM
χPT	χPT	Special Galileon	Born-Infeld
YM	YM	Born-Infeld	Gravity

[Cachazo, He Yuan]

We are interested in

$$\mathcal{A}_4^{\chi\text{PT}}[1234] = \frac{t}{s}\mathcal{A}_4^{\chi\text{PT}}[1324]$$

We can feed this into the double copy

$$\text{BI} = \text{YM} \otimes \chi\text{PT}$$

$$\text{e.g.} \quad \mathcal{A}_4^{\text{BI}}(1234) = \frac{us}{t}\mathcal{A}_4^{\text{YM}}[1234]\mathcal{A}_4^{\chi\text{PT}}[1234]$$

Amplitude "Bootstrap"

Central idea: The form of the amplitude is fully or partially fixed upon imposing a given list of physical conditions:

- Locality: Analytic structure
- Unitarity: Positivity bounds and intermediate particle constraints
- Lorentz covariance: Must obey little group scaling

Additionally symmetries or constraints:

- Soft behaviour: For spontaneously broken symmetries
- Collinear behaviour: For gluons and gravitons
- Good UV behaviour: For renormalizable theories or special shifts

Let us see an example of the soft bootstrap for pion EFT.

Soft Amplitudes

Recall: Amplitudes of Goldstone modes (like pions) vanish in the limit of vanishing external momenta,

$$\lim_{p_i \to 0} \mathcal{A}_n(1, 2, \cdots, n) = 0\,.$$

Is this enough to fix NLSM amplitudes?

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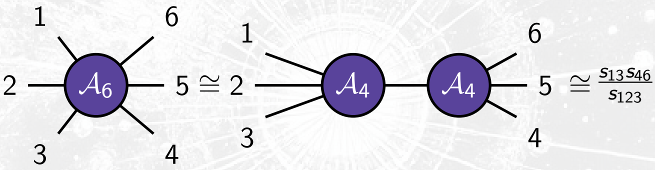
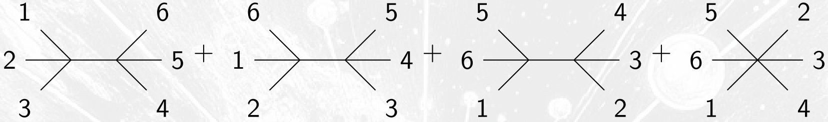
Locality tells us where the poles are and how the pole terms look



where $s_{ij} = (p_i + p_j)^2$.

Example: Soft Constraints in NLSM

Add known pole terms to an ansatz for terms with no poles,



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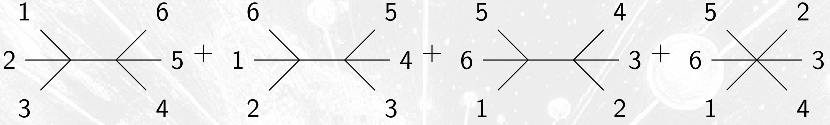


Taking the soft limit should vanish,

$$\lim_{p_1 \to 0} \left(\frac{s_{13}s_{46}}{s_{123}} + \frac{s_{35}s_{26}}{s_{126}} + \frac{s_{15}s_{24}}{s_{156}} + \text{contact term} \right)$$
$$= s_{35} + \lim_{p_1 \to 0} \text{contact term} = 0$$

Applying this condition for the soft limit of all the momenta, uniquely fixes the contact term,

$$\text{contact term} = s_{135}$$

This happens at every n-point. Is this surprising?

BCJ Bootstrap

Locality + BCJ: No simple poles at $t = 0$

Unitarity + soft behaviour: No simple poles

$$\mathcal{A}_4[1234] = f(s,t) = c_0 + \frac{c_{11}s + c_{12}t}{\Lambda^2} + \frac{c_{21}s^2 + c_{22}st + c_{23}t^2}{\Lambda^4} + O\left(\frac{1}{\Lambda^6}\right)$$

Cyclicity + relabeling: $f(s,t) = f(-s-t,s)$

KK + BCJ + relabeling: $s\,f(s,t) = t\,f(t,s)$

$$\mathcal{A}_4[1234] = \frac{t}{\Lambda^2}\left(1 + \frac{c_4}{\Lambda^4}(s^2+t^2+u^2) + \frac{c_6}{\Lambda^6}stu + O\left(\frac{1}{\Lambda^8}\right)\right)$$

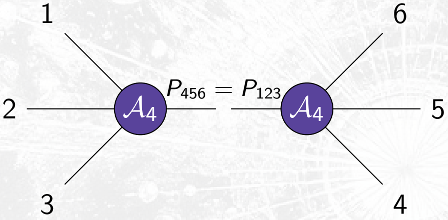
$$\mathcal{A}_4[1234] = \frac{t}{\Lambda^2}\,(\text{symmetric polynomial in } s,\ t,\ u)$$

Corrections at 6-Point

Order of constraints:

$$\begin{array}{rcl} \text{Unitarity} & \longrightarrow \text{Cyclicity} & \longrightarrow \text{BCJ} \\ \text{Construct pole terms} & \longrightarrow \text{Add cyclic contact terms} & \longrightarrow \text{BCJ} \end{array}$$

To construct pole terms, we glue together 4-point amplitudes:



$+$ cyclic $+$ contact terms

Ambiguities in taking $\mathcal{A}_4$ off-shell are taken care of by adding all possible contact terms.

BCJ Constraints in NLSM

	Derivatives	BCJ terms	Contact	Poles
4-point	0	0	0	
	2	1	1	NLSM leading order
	4	0	0	
	6	1	1	
	8	1	1	
	10	1	1	
	12	1	1	
	14	2	2	
6-point	0	0	0	No poles
	2	1 $\mathcal{O}(p)$	0	$\partial^2 \times \partial^2$
	4	0	0	No poles
	6	1 $\mathcal{O}(p)$	0	$\partial^2 \times \partial^6$
	8	1 $\mathcal{O}(p)$	0	$\partial^2 \times \partial^8$
	10	2 $\mathcal{O}(p)$	0	$\partial^2 \times \partial^{10} + \partial^6 \times \partial^6$
	12	4 $\mathcal{O}(p)$	2	$\partial^2 \times \partial^{12} + \partial^6 \times \partial^8$
	14	7 $\mathcal{O}(p)$	4	$\partial^2 \times \partial^{14} + \partial^6 \times \partial^{10} + \partial^8 \times \partial^8$

All BCJ terms have soft behaviour. Not all soft amplitudes are BCJ compatible. So what properties do BCJ terms have?

BCJ vs. Soft Constraints in NLSM

Constraint	Order	Result	Relation to BCJ
Adjacent double soft	$\mathcal{O}(p^4)$ at 6-pt		No constraints
	$\mathcal{O}(p^4)$ at 8-pt		No constraints
	$\mathcal{O}(p^6)$ at 6-pt		No constraints
	$\mathcal{O}(p^6)$ at 8-pt		No constraints
Next-to-adjacent double soft	$\mathcal{O}(p^4)$ at 6-pt	$L_3 = 0$	Partial 4-pt BCJ
	$\mathcal{O}(p^4)$ at 8-pt		Partial 4-pt BCJ
	$\mathcal{O}(p^6)$ at 6-pt	$L_3 = 0$	Partial 4-pt BCJ
	$\mathcal{O}(p^6)$ at 8-pt		Partial 4-pt BCJ
Kleiss-Kuif + extended theory	$\mathcal{O}(p^4)$ at 6-pt	$L_3 = L_0 = 0$	Full 4-pt BCJ
	$\mathcal{O}(p^4)$ at 8-pt		Full 4-pt BCJ
	$\mathcal{O}(p^6)$ at 6-pt	$C_{62} = \frac{1}{2} C_{61}$, $C_{62} = -\frac{2}{3} C_{61}$ $C_{64} = 0$, $C_{65} = C_{61}$ $C_{66} = -2C_{61}$, $C_{67} = \frac{5}{3} C_{61}$	Full 6-pt BCJ

Interestingly, the existence of the extended theory places the same constraints as BCJ.

Results

Soft and BCJ bootstrap results at 4-point

$\mathcal{O}(p^{\#})$	2	4	6	8	10	12	14	16	18
Soft amplitudes	1	2	3	4	5	6	7	8	9
BCJ amplitudes	1	0	1	1	1	1	2	1	2

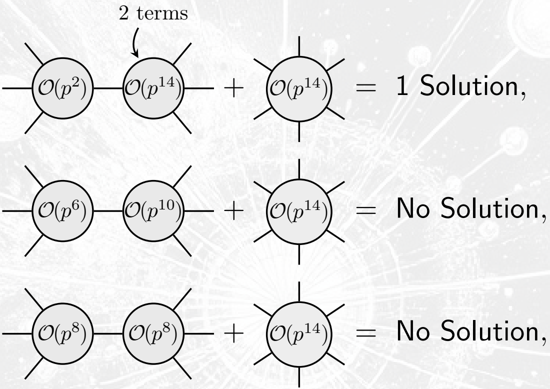
Soft and BCJ bootstrap results at 6-point

$\mathcal{O}(p^{\#})$	2	4	6	8	10	12	14	16	18
Soft amplitudes	1	2	10	29	83	207	461	945	1819
- Contact terms	0	0	5	22	70	191	434	915	1772
BCJ amplitudes	1	0	1	1	2	4	7	16	36
- Contact terms	0	0	0	0	0	2	4	13	31

[Brown, Kampf, Oktem, SP, Trnka]

Interesting Feature 1: 4-Point Selection

The first order at which we get two different 4-point amplitudes is ∂^{14}. Here something interesting happens.



Unlike with soft behaviour, 4-point BCJ does not extend to 6-point BCJ.

Interesting Feature 2: Z-Theory

Open string amplitudes can be factorized into

$$\text{open string} \; = \; \text{SYM} \; \otimes \; \text{Z-theory}$$

i.e. into a QFT building block and a disk integral.

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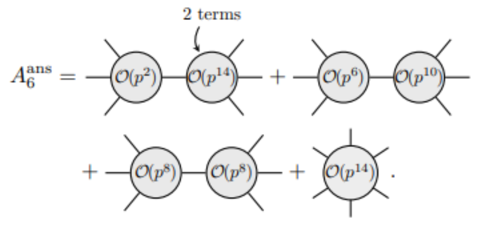
At 4-point, not all possible BCJ corrections are generated. Only this selected combination is.



Chiral vs. BCJ Perturbation Theory

Why does chiral perturbation theory work?

Because the chiral Lagrangian (or equivalently the resulting amplitude) can be fixed order by order in both derivatives and multiplicity.

We now see that color-kinematic constraints work differently. There are cross-derivative order and cross-multiplicity constraints.

As we include higher-order and higher-n constraints, does the NLSM get closer to the low-energy limit of Z-integrals?

Experimental Bounds on Coefficients

A comparison can be done to Low Energy Constant (LEC) determinations from scattering data.

$$\begin{aligned}\mathcal{L}_4^{n_f=3} &= L_1\langle u_\mu u^\mu\rangle\langle u_\nu u^\nu\rangle + L_2\langle u_\mu u_\nu\rangle\langle u^\mu u^\nu\rangle + L_3\langle u_\mu u^\mu u_\nu u^\nu\rangle \\ &+L_4\langle u_\mu u^\mu\rangle\langle\chi_+\rangle + L_5\langle u_\mu u^\mu \chi_+\rangle + L_6\langle\chi_+\rangle^2 + L_7\langle\chi_-\rangle^2 \\ &+\frac{L_8}{2}\langle\chi_+^2+\chi_-^2\rangle - iL_9\langle f_{+\mu\nu}u^\mu u^\nu\rangle + \frac{L_{10}}{4}\langle f_+^2 - f_-^2\rangle \\ &+\ 2 \text{ contact terms} .\end{aligned}$$

And the fits for the NLO coefficients are

	free fit				p^4 fit	Ref. [3]
$10^3 L_A^r$	1.17(27)	0.39(16)	0.52(16)	0.27(16)	0.4(2)	
$10^3 L_1^r$	1.11(10)	0.98(09)	1.00(09)	0.95(09)	1.0(1)	0.7(3)
$10^3 L_2^r$	1.05(17)	1.56(09)	1.48(09)	1.64(09)	1.6(2)	1.3(7)
$10^3 L_3^r$	−3.82(30)	−3.82(30)	−3.82(30)	−3.82(30)	−3.8(3)	−4.4(2.5)
$10^3 L_4^r$	1.87(53)	$\equiv 0$	$\equiv 0.3$	$\equiv -0.3$	0.0(3)	−0.3(5)
$10^3 L_5^r$	1.22(06)	1.23(06)	1.23(06)	1.23(06)	1.2(1)	1.4(5)
$10^3 L_6^r$	1.46(46)	−0.11(05)	0.14(06)	−0.36(05)	0.0(4)	−0.2(3)
$10^3 L_7^r$	−0.39(08)	−0.24(15)	−0.27(14)	−0.21(17)	−0.3(2)	−0.4(2)
$10^3 L_8^r$	0.65(07)	0.53(13)	0.55(12)	0.50(14)	0.5(2)	0.9(3)
χ^2	3.8	16	12	20		
dof	4	5	5	5	5	
F_0 [MeV]	58	81	76	86	81(5)	

[Bijnens, Ecker][Gasser, Leutwyler][Alvarez, Bijnens, Sjo]

Other Examples of EFT applications

- An extension of the KLT kernel and BCJ relations in order to encompass a wider range of higher-derivative corrections.

  [Chi, Elvang, Herderschee, Jones, SP]

- A construction of seeds for numerators at 4- and 5-point to get higher-derivative corrections to YM, NLSM and gravity.

  [Carrasco, Rodina, (Yin), Zekioglu]

- Establishing a connection between KLT and BCJ formalisms at higher-derivative and solve for higher-point seeds.

  [Bonnefoy, Durieux, Grojean, Machado, Nepveu]

- Color-Kinematics can be extended to NLSMs of non-symmetric cosets

  [Helset]

- Double copying full non-perturbative S-matrices of scalars (with masses and higher-derivative corrections) in $1+1$ dimensions

  [Cheung, Mangan, Parra-Martinez, Shah]

Adding Mass

So far, everything has been massless. Time to add masses!

But first- let us understand why adding mass is so challenging.

The KLT relation can be written as

$$M_n = \sum_{\alpha,\beta} S[\alpha|\beta] A_n[\alpha] A_n[\beta]$$

where the KLT kernel $S[\alpha|\beta]$ is a rational function of Mandelstam invariants that happens to be the inverse of a matrix of bi-adjoint scalar amplitudes $S[\alpha|\beta] = m_n[\alpha|\beta]^{-1}$.

One way to generalize the double copy, is to add mass to the KLT kernel and BCJ relations by first adding mass to bi-adjoint scalar amplitudes.

Cubic Massless Bi-adjoint Scalar Theory

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + g\ f_{abc} f_{a'b'c'} \phi^{aa'} \phi^{bb'} \phi^{cc'} + m^2\phi^2$$

- Doubly-color ordered tree amplitudes

$$\mathcal{A}_n(1^{a_1 a'_1} \ldots n^{a_n a'_n}) = \sum_{\sigma,\gamma} \mathcal{A}_n[1\gamma|1\sigma] \mathrm{Tr}\left[T^{a_1} T^{\gamma(a_2} \ldots T^{a_n)}\right] \times \mathrm{Tr}\left[T^{a'_1} T^{\sigma\left(a'_2\right.} \ldots T^{\left.a'_n\right)}\right]$$

- Kleiss-Kuijf relations $\Rightarrow$ $(n-2)!$ distinct color-orderings
- BCJ relations $\Rightarrow$ $(n-2)! \times (n-2)!$ matrix of amplitudes has rank $(n-2)!$

Spurious Poles

At 5-point, the kernel with all-identical masses has non-physical singularities:

$$
\begin{aligned}
& m^8 \bigg(s_{12}^4 + \left(2s_{13} + 2s_{14} + 2s_{23} + 2s_{24} - 6m^2\right) s_{12}^3 + \left(s_{13}^2 + 2s_{13}s_{14} + 2s_{13}s_{23} + 4s_{13}s_{24} + s_{14}^2 + 4s_{14}s_{23} + 2s_{14}s_{24} + s_{23}^2\right. \\
&\quad \left. + 2s_{23}s_{24} + s_{24}^2 - 6m^2 s_{13} - 6m^2 s_{14} - 6m^2 s_{23} - 6m^2 s_{24} + m^4\right) s_{12}^2 + \left(2s_{13}^2 s_{24} + 2s_{13}s_{14}s_{23} + 2s_{13}s_{14}s_{24} + 2s_{13}s_{23}s_{24}\right. \\
&\quad + 2s_{13}s_{24}^2 + 2s_{14}^2 s_{23} + 2s_{14}s_{23}^2 + 2s_{14}s_{23}s_{24} + 4m^2 s_{13}s_{14} - 4m^2 s_{13}s_{23} - 6m^2 s_{13}s_{24} - 6m^2 s_{14}s_{23} - 4m^2 s_{14}s_{24} \\
&\quad \left. + 4m^2 s_{23}s_{24} - 8m^4 s_{13} - 8m^4 s_{14} - 8m^4 s_{23} - 8m^4 s_{24} + 24m^6\right) s_{12} + s_{13}^2 s_{24}^2 - 2s_{13}s_{14}s_{23}s_{24} + s_{14}^2 s_{23}^2 + 4m^2 s_{13}^2 s_{14} \\
& + 4m^2 s_{13}s_{14}^2 + 4m^2 s_{13}s_{14}s_{23} + 4m^2 s_{13}s_{14}s_{24} + 4m^2 s_{13}s_{23}s_{24} + 4m^2 s_{14}s_{23}s_{24} + 4m^2 s_{23}^2 s_{24} + 4m^2 s_{23}s_{24}^2 - 8m^4 s_{13}^2 \\
& - 20m^4 s_{13}s_{14} - 8m^4 s_{13}s_{23} - 8m^4 s_{13}s_{24} - 8m^4 s_{14}^2 - 8m^4 s_{14}s_{23} - 8m^4 s_{14}s_{24} - 8m^4 s_{23}^2 - 20m^4 s_{23}s_{24} - 8m^4 s_{24}^2 \\
& + 24m^6 s_{13} + 24m^6 s_{14} + 24m^6 s_{23} + 24m^6 s_{24} - 20m^8 \bigg) .
\end{aligned}
$$

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$$m^8\Big(s_{12}^4+\left(2s_{13}+2s_{14}+2s_{23}+2s_{24}-6m^2\right)s_{12}^3+\big(s_{13}^2+2s_{13}s_{14}+2s_{13}s_{23}+4s_{13}s_{24}+s_{14}^2+4s_{14}s_{23}+2s_{14}s_{24}+s_{23}^2$$
$$+2s_{23}s_{24}+s_{24}^2-6m^2s_{13}-6m^2s_{14}-6m^2s_{23}-6m^2s_{24}+m^4\big)s_{12}^2+\big(2s_{13}^2s_{24}+2s_{13}s_{14}s_{23}+2s_{13}s_{14}s_{24}+2s_{13}s_{23}s_{24}$$
$$+2s_{13}s_{24}^2+2s_{14}^2s_{23}+2s_{14}s_{23}^2+2s_{14}s_{23}s_{24}+4m^2s_{13}s_{14}-4m^2s_{13}s_{23}-6m^2s_{13}s_{24}-6m^2s_{14}s_{23}-4m^2s_{14}s_{24}$$
$$+4m^2s_{23}s_{24}-8m^4s_{13}-8m^4s_{14}-8m^4s_{23}-8m^4s_{24}+24m^6\big)s_{12}+s_{13}^2s_{24}^2-2s_{13}s_{14}s_{23}s_{24}+s_{14}^2s_{23}^2+4m^2s_{13}^2s_{14}$$
$$+4m^2s_{13}s_{14}^2+4m^2s_{13}s_{14}s_{23}+4m^2s_{13}s_{14}s_{24}+4m^2s_{13}s_{23}s_{24}+4m^2s_{14}s_{23}s_{24}+4m^2s_{23}^2s_{24}+4m^2s_{23}s_{24}^2-8m^4s_{13}^2$$
$$-20m^4s_{13}s_{14}-8m^4s_{13}s_{23}-8m^4s_{13}s_{24}-8m^4s_{14}^2-8m^4s_{14}s_{23}-8m^4s_{14}s_{24}-8m^4s_{23}^2-20m^4s_{23}s_{24}-8m^4s_{24}^2$$
$$+24m^6s_{13}+24m^6s_{14}+24m^6s_{23}+24m^6s_{24}-20m^8\Big)\,.$$

Where did these come from?

$$S_{\mathrm{KLT}}[\alpha|\beta]=m[\alpha|\beta]^{-1}=\frac{1}{\det m}(\text{matrix of cofactors})$$

where m is a matrix of amplitudes i.e. contains no spurious singularities.

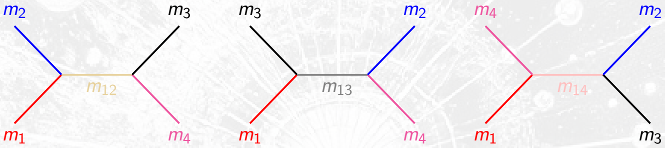
$\Rightarrow$ The only source of spurious poles is the numerator of $\det m$.

[Johnson, Jones, SP]

Possible Ways Out

Minimal rank conjecture: Any $m_n[\alpha|\beta]$ with rank $(n-3)!$ will lead to a local double copy.

At 4-point, add masses to bi-adjoint scalar theory in the most generic fashion:

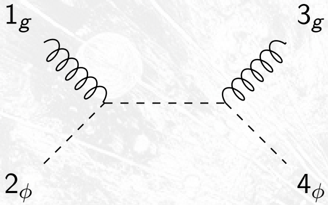


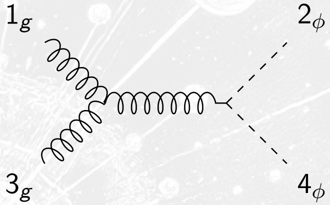
$\Rightarrow m_1^2 + m_2^2 + m_3^2 + m_4^2 = m_{12}^2 + m_{13}^2 + m_{14}^2.$

[Johnson, Jones, SP]

Satisfying the Spectral Conditions: Special Amplitudes

Compton scattering: $g + \phi \to g + \phi$





$$m^2 + m^2 + 0^2 = m^2 + m^2 + 0^2 + 0^2$$

[Johansson, Ochirov]

Satisfying the Spectral Conditions: KK Tower

- Theories that result from dimensional reduction e.g. on S^1 have vertices that conserve mass
- For a process 1+2→3+4,

$$m_{12} = m_1 + m_2 \qquad m_{13} = m_1 - m_3$$
$$m_{14} = m_1 - m_4 \qquad m_1 + m_2 = m_3 + m_4$$
$$\Rightarrow m_{12}^2 + m_{13}^2 + m_{14}^2 = m_1^2 + m_2^2 + m_3^2 + m_4^2$$

- So any BCJ-compatible theory with a Kaluza-Klein tower of states can be double-copied.

[Johnson, Jones, SP]

Color-kinematic mYM is fixed to KK compactification of 5d YM

[Momeni, Rumbutis, Tolley][Gonzalez, Liang, Trodden]

⇒ KK mYM can be double-copied to a local theory of massive gravity.

QCD and the Double Copy

- Color-kinematics duality holds (in a modified sense) for tree amplitudes with gluons and quark-antiquark pairs (i.e. fundamental matter).

  [Johansson, Ochirov]

- Up to 1-loop 5-points, massive scalar QCD with any kind of matter exhibits color-kinematic duality.

  [Plefka, Shi, Wang]

- Super-QCD integrands and amplitudes are color-kinematics dual up to 2-loop.

  [Duhr, Johansson, Kalin, Mogull, Verbeek]

- Higgsed YM and SYM with broken SUSY can also be double-copied to get gauged supergravities.

  [Chiodaroli, Gunaydin, Johansson, Roiban]

Other Examples with Mass

- Calculate top tree amplitudes using BCFW and reduce number of "primitives" via BCJ.

  [Campbell, Ellis]

- Color-kinematics can be used at tree and 1-loop level to calculate massive fermions coupled to higher-derivative YM.

  [Menezes]

- Topologically massive theories satisfy BCJ relations.

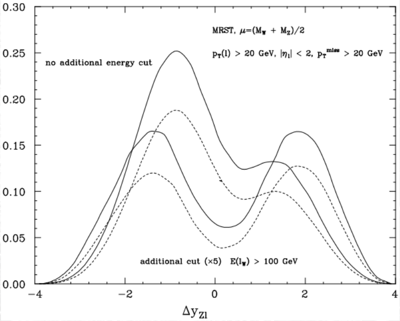
  [Gonzalez, Momeni, Rumbutis]

- HQET shows color-kinematic duality up to 4-point, double copying to HBET.

  [Haddad, Helset]

Radiation zeros

- Di-boson production from quark annihilation has "radiation zeros" at leading order.
  [Brown, Mikaelian, (Sahdev)][Mikaelian, Samuel, Sahdev][Baur, Han, Ohnemus]
- Take $q_1 \bar{q}_2 \to W^{\pm} Z$ for example:



[Dixon, Kunszt, Signer]

Hidden Zeros

Recently, hidden zeros were discovered in partial amplitudes of a certain class of theories e.g. NLSM, Yang-Mills and $\mathrm{Tr}(\phi^3)$.

[Arkani-Hamed, Cao, Dong, Figueiredo, He]

In d-dimensional scalar theories, these zeros are reached by sending a specific set of Mandelstam invariants $s_{ij} = (p_i + p_j)^2$ to zero,

4-point	$s_{13} = 0$	
5-point	$s_{13} = s_{14} = 0$	
6-point	$s_{13} = s_{14} = s_{15} = 0$	C_2
	$s_{14} = s_{15} = s_{24} = s_{25} = 0$	C_3

Note: The first type of zero is one Mandelstam away from being an Adler zero $p_1 \to 0$.

Examples in SU(N) NLSM

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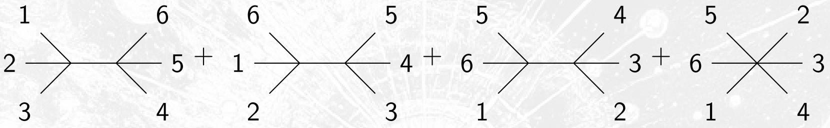

$$A_6^{\mathrm{NLSM}}[123456] = \frac{s_{13}s_{46}}{s_{123}} + \frac{s_{35}s_{26}}{s_{126}} + \frac{s_{15}s_{24}}{s_{156}} - s_{135}$$

The zeros are not manifest.

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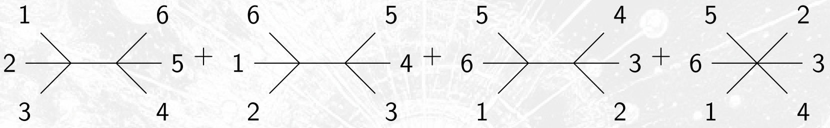

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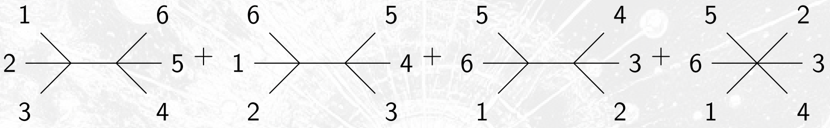

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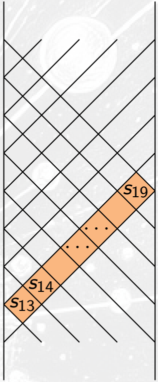
General n-Point Example

For scalar theories like NLSM and $\mathrm{Tr}(\phi^3)$, $A_n|_{C_m} = 0$ where the zero condition is given by

$$C_m = \begin{cases} s_{1m+1} = s_{1m+2} = \cdots = s_{1n-1} = 0 \\ s_{2m+1} = s_{2m+2} = \cdots = s_{2n-1} = 0 \\ \vdots \\ s_{m-1m+1} = \cdots = s_{m-1n-1} = 0 \end{cases}$$

where $m = 2, \cdots, \lfloor \frac{n}{2} \rfloor$.



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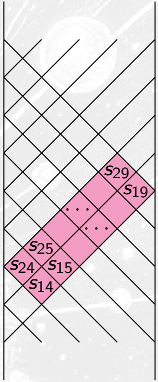

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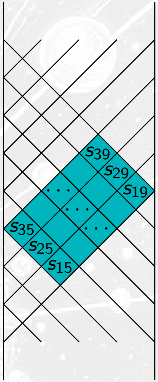

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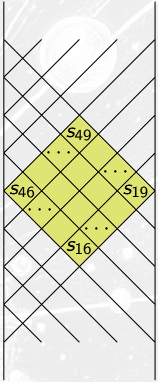
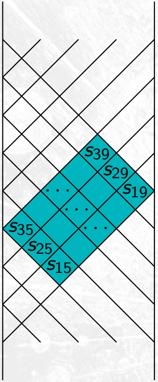


Factorization Near Zeros

Like near poles, the amplitude factorizes near zeros into lower-point amplitudes, when all-but-one Mandelstam in C_m vanishes.



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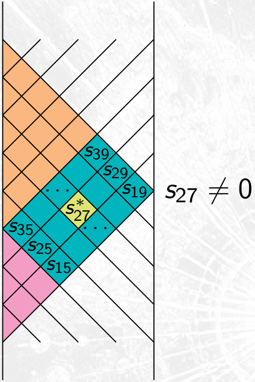

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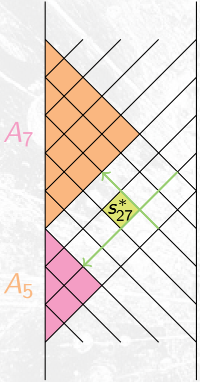

$$A_{10} \xrightarrow{C_4, s^*_{27}} \left(\frac{1}{s_{1234}} + \frac{1}{s_{3456789}}\right) \times A_5 \times A_7$$

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Spinning Zeros

For gluons, we need to extend the zero conditions slightly to now include polarization vectors as well i.e.

$$\text{if } s_{ij} = 0 \text{ on } C_m\,,$$
$$(p_i \cdot p_j) = (\varepsilon_i \cdot p_j) = (p_i \cdot \varepsilon_j) = (\varepsilon_i \cdot \varepsilon_j) = 0 \text{ on } C_m^{\text{spinning}}$$

4-point YM has contact+pole terms:

$$(\varepsilon_1 \cdot \varepsilon_3)(\varepsilon_2 \cdot \varepsilon_4) \xrightarrow{(\varepsilon_1 \cdot \varepsilon_3) \to 0} 0$$

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$$\frac{1}{s_{12}}\left((\varepsilon_1 \cdot \varepsilon_2)p_1^\mu + (\varepsilon_1 \cdot p_2)\varepsilon_2^\mu + (\varepsilon_2 \cdot p_1)\varepsilon_1^\mu\right)$$
$$\times \left((\varepsilon_3 \cdot \varepsilon_4)p_3^\nu + (\varepsilon_3 \cdot p_4)\varepsilon_4^\nu + (\varepsilon_4 \cdot p_3)\varepsilon_3^\nu\right)\eta_{\mu\nu} + \text{cyc} \xrightarrow{C_2} 0$$

Can this be seen from any of the many constructions we have for YM amplitudes?

Theories with Hidden Zeros

So far, we've seen that the following theories have hidden zeros:

- Tr(ϕ^3) theory of adjoint scalars
- SU(N) non-linear sigma model
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They all "play a role" in the double copy.

Hidden Zeros and the Double Copy

It turns out that

- Color-kinematics / BCJ relations $\Rightarrow$ hidden zeros and factorization (assuming good UV behaviour)
- Double copy / KLT relations $\Rightarrow$ hidden zeros and factorization (assuming good UV behaviour)

[Bartsch, Brown, Kampf, Oktem, SP, Trnka][Jones, SP (to appear)]

Does this extend to other types of radiation zeros? Are these types of zeros or structures present in other amplitudes?

Further Scope

- Positivity bounds+BCJ lands you closer and closer to the stringy completion of NLSM.

  [Carrasco, Pavao][Chen, Herderschee, Elvang]

- Form factor BCJ-like relations and double copy: In scalar-YM theory, form factors of Tr(ϕ^2) exhibit color-kinematic form. Also in QCD with a Higgs with operator a quark bilinear.

  [Lin, Yang]

- Celestial amplitudes of gluons exhibit color-kinematic duality and a double copy

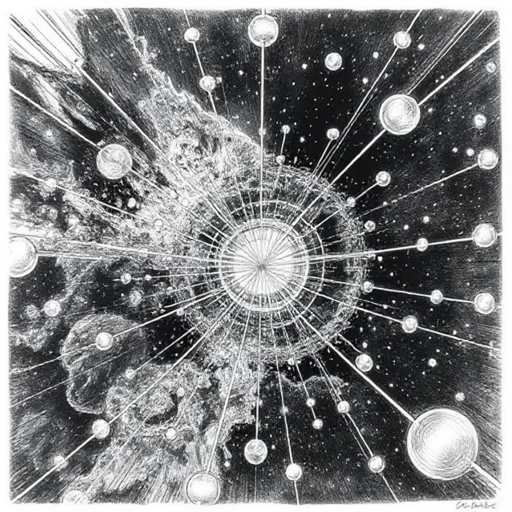
  [Casali, Puhm][Monteiro][Guevara]

- Classical solutions of anyons

  [Burger, Emond, Moynihan]

Last Remarks

- The double copy is a powerful tool for understanding gravity.
- On the other hand, double-copy-ability or color-kinematics duality is a strong constraining principle.
- One can use it as a bootstrap condition either for higher-derivative or massive theories.
- A large class of interesting theories exhibit BCJ-compatibility, including QCD and pion EFTs.
- Further, you can use it to construct loop integrands, classical solutions, form factors, etc.



Thank You