

Scalar Amplitudes at Infinity



Shruti Paranjape
Brown University

Discussions on Quantum Spacetime
Lodha Mathematical Sciences Institute, Mumbai

Based on [arXiv:2504.11253](https://arxiv.org/abs/2504.11253) with M Skowronek, M Spradlin, A Volovich
and [arXiv:2505.02520](https://arxiv.org/abs/2505.02520) with C Jones

Tree Amplitudes

At tree-level, the amplitude is a rational function

$$\mathcal{A}_n = \frac{\text{numerator}}{\text{denominator}}$$

Additionally, a lot is known about the singularity structure from:

- **Locality:** Poles and branch cuts of the amplitudes

$$\lim_{p_I^2 \rightarrow 0} \mathcal{A}_n \sim \frac{1}{p_I^2}$$

- **Unitarity:** Factorization into lower-point amplitudes

$$\text{Res}_{p_I^2=0} \mathcal{A}_n = \mathcal{A}_L \times \mathcal{A}_R$$



Numerators

Unitarity



n -point numerator at **singular** kinematic points



Determines the amplitude up to “off-pole” contributions

Zeros and poles fully determine any rational function $\Rightarrow$ do amplitudes have **zeros**?

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Symmetry	Zero	Example Theory
$\phi \rightarrow \phi + c$	$\mathcal{A}_n \xrightarrow{p_i \rightarrow 0} \mathcal{O}(p_i)$	$\frac{SU(N) \times SU(N)}{SU(N)}$ NLSM
$\phi \rightarrow \phi + c + a^\mu x_\mu$	$\mathcal{A}_n \xrightarrow{p_i \rightarrow 0} \mathcal{O}(p_i^2)$	DBI brane scalar
$\phi \rightarrow \phi + c + a^\mu x_\mu + s^{\mu\nu} x_\mu x_\nu$	$\mathcal{A}_n \xrightarrow{p_i \rightarrow 0} \mathcal{O}(p_i^3)$	special Galileon theory

Numerators

Do amplitudes have poles at **infinity**? This is related to recursibility.

Parameterize an approach to infinity by a complex parameter z :

$$p_i^\mu \rightarrow \hat{p}_i^\mu(z), \quad \mathcal{A}_n(p_i) \rightarrow \mathcal{A}_n(\hat{p}_i), \quad \text{and then take } z \rightarrow \infty.$$

Set up a integral:

$$\mathcal{A}_n(z=0) = \oint_{C_0} \frac{dz}{z} \mathcal{A}_n(z) = 2\pi i \sum_{z=z_i} \text{Res} \frac{1}{z} \mathcal{A}_n(z) + 2\pi i \text{Res}_{z=\infty} \frac{1}{z} \mathcal{A}_n(z).$$

Poles at **finite** locations are well-understood.

- **Locality** $\Rightarrow$ poles when intermediate momenta go on-shell.
- **Unitarity** $\Rightarrow$ residue is the factorized amplitude.

Thus, **if** the amplitude has no pole at infinity, it is given by lower-point amplitudes.

Behaviour at infinity

There are many **different on-shell directions of approach** to infinity.

Some have poles at infinity, some do not.

Example for color-ordered Yang-Mills amplitudes under a BCFW shift:

i, j	Behaviour
$1^+ 2^+$	$1/z$
$1^- 2^+$	$1/z$
$1^- 2^-$	$1/z$
$1^+ 2^-$	$1/z^2$
$1^+ 3^+$	$1/z^2$

$$|i] = |i\rangle + z|j],$$

$$|j\rangle = |j\rangle - z|i\rangle.$$

Does the function at infinity have any factorization properties? Is it predictable? Is it an amplitude of a different theory?

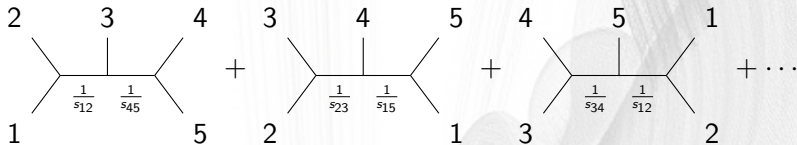
$$\mathcal{A}_n(z) \sim z^\# \mathcal{A}_n^\infty$$

Good Directions and Feynman Diagrams

The main theory of interest is $\text{Tr } \phi^3$

$$\mathcal{L} = \text{Tr}(\phi_{ab}\phi_{bc}\phi_{ca})$$

The 5-point amplitude has many Feynman diagrams



For the amplitude to have **no pole at infinity**, we must take the propagators to infinity in a direction such that

$$\mathcal{A}_n \xrightarrow[z \rightarrow \infty]{s_i \rightarrow s_i + a_i z} \mathcal{O}\left(\frac{1}{z^2}\right)$$

Kinematic Variables in Planar Scattering

Color-ordered amplitudes lend themselves to a nice kinematic variables:

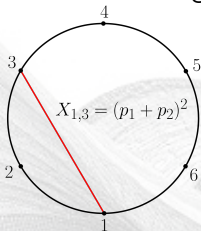
$$X_{ij} = (p_i + \cdots p_{j-1})^2$$

Massless $\Rightarrow X_{ii+1} = 0$ and **momentum conservation** $\Rightarrow X_{ij} = X_{ji}$. These are independent in high enough dimension.

There are also non-planar variables $c_{ij} = (p_i + p_j)^2$ which satisfy:

$$c_{ij} = X_{ij} + X_{i+1j+1} - X_{ij+1} - X_{i+1j}.$$

Planarity allows us to organize X 's on a surface:



- Every triangulation is a **valid Feynman diagram**.
- Factorization **splits** the surface in two.

g -vector Shifts

Choose a basis of kinematic variables containing

- $X_{ij} \in \mathcal{T}$: $n-3$ chords that form a **triangulation**.
- $c_{ij} \in \mathcal{T}$ if $X_{i+1j+1} \notin \mathcal{T}$: All but $n-3$ non-planar variables complete the basis.

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We now consider a shift $(\mathcal{T}, \vec{t})$ where

- $X_{ij} \rightarrow X_{ij} + \vec{t}_{ij}z$ for all X in the basis.
- $c_{ij} \in$ basis do **not** shift.
- X_{ij} not in basis shift such that the c_{ij} can remain unshifted.

This is an alternate way of saying $X \rightarrow X + \vec{g}_X \cdot \vec{t}z$ where $\vec{g}_X$ is the vector associated to X in the **Feynman fan** of a base triangulation.

[Arkani-Hamed, Frost, Salvatori, Plamondon, Thomas]

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[Arkani-Hamed, Frost, Salvatori, Plamondon, Thomas]

Projective invariance of the $\text{Tr } \phi^3$ scattering form means that

$$\mathcal{A}_n(z) \xrightarrow{\text{g-vector shift}} \mathcal{O}\left(\frac{1}{z^2}\right)$$

[Arkani-Hamed, Bai, He, Yan]

4-point Example

- Choose a triangulation of a square: X_{13} .
- Fill the rest of the basis up with c 's: c_{13} .
- Look at a specific shift $X_{13} \rightarrow X_{13} + z$.

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The other variables in this basis are

$$X_{24} = c_{13} - X_{13}$$

$$c_{24} = c_{13}$$

Thus under the shift $X_{24} = X_{24} - z$ and the amplitude becomes

$$\hat{\mathcal{A}}_4 = \frac{1}{\hat{X}_{13}} + \frac{1}{\hat{X}_{24}} = \frac{X_{13} + X_{24}}{\hat{X}_{13}\hat{X}_{24}} \xrightarrow{z \rightarrow \infty} \frac{1}{z^2} c_{13}$$

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[Arkani-Hamed, Cao, Dong, Figueiredo, He]

Note that this is a **4-pion** NLSM amplitude. At every n -point, there are special g -vector shifts that give rise to pions.

5-point Example

- Choose a triangulation of a pentagon: X_{13}, X_{14} .
- Fill the rest of the basis up with c 's: c_{13}, c_{14}, c_{24} .
- Look at a specific shift $X_{13} \rightarrow X_{13} + z, X_{14} \rightarrow X_{14} - z$.

5-point Example

- Choose a triangulation of a pentagon: X_{13}, X_{14} .
- Fill the rest of the basis up with c 's: c_{13}, c_{14}, c_{24} .
- Look at a specific shift $X_{13} \rightarrow X_{13} + z, X_{14} \rightarrow X_{14} - z$.

The other X 's can be expressed in this basis:

$$\begin{aligned} X_{24} &= c_{13} - X_{13} + X_{14}, & X_{25} &= c_{13} + c_{14} - X_{13}, & X_{35} &= c_{14} + c_{24} - X_{14}, \\ c_{25} &= X_{25} + X_{13} - X_{35}, & c_{35} &= X_{35} + X_{14} - X_{13}. \end{aligned}$$

Thus under the shift:

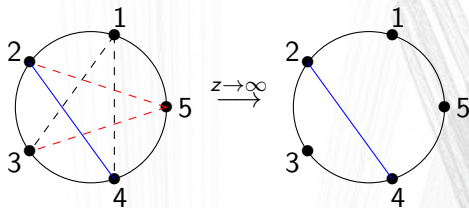
$$X_{24} \rightarrow X_{24}, X_{25} \rightarrow X_{25} - z, X_{35} \rightarrow X_{35} + z, c_{25} \rightarrow c_{25} - z, c_{35} \rightarrow c_{35} - z.$$

Taking $z \rightarrow \infty$, the amplitude is

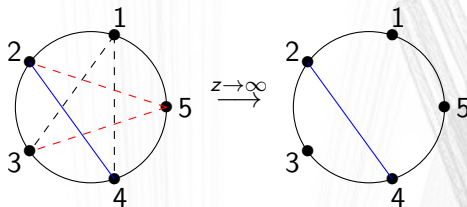
$$\mathcal{A}_5 \xrightarrow{z \rightarrow \infty} \frac{1}{z^2} \frac{c_{14}}{X_{24}}$$

Can we predict this function?

Shifting the Surface



Shifting the Surface



Thus the function at infinity must look like:

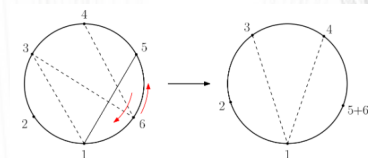
$$\frac{1}{z^2} \frac{\text{numerator of } (234) \times \text{numerator of } (1245)}{X_{24}}$$

- Dimensional analysis tells us that the numerator has **dimension 1**
- Kinematic surface tells us that the **numerator factorizes**
- Projective invariance of the scattering form on subsurfaces tells us the **overall z scaling**

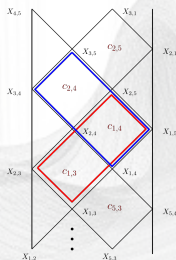
Assigning Numerators

Numerators can be assigned to subsurfaces via the following procedure:

- Base triangulation of subsurface is **inherited** from the full surface:



- Actual numerator can be read off from the kinematic mesh by drawing **causal diamonds**:



Final Formula

$$\mathcal{A}_n^\infty = \prod_{\substack{S_i \subset S \\ |S_i| \geq 4}} \frac{1}{z^{|S_i|-2}} \prod_{S_i \subset S} \frac{1}{\text{PT}_{S_i}} \sum_{\mathcal{T}_{\text{partial}}} \left(\prod_{i,j \in \mathcal{T}_{\text{partial}}} \frac{1}{x_{i,j}} \right) \prod_{\substack{S_i \subset S \\ n_i \geq 4}} c_{S_i}$$

[SP, Skowronek, Spradlin, Volovich]

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Product of factors of $1/z$ for every subsurface **larger than a triangle**. Such surfaces involve a propagator that has been sent to infinity.

[SP, Skowronek, Spradlin, Volovich]

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Product of **Parke-Taylor factors** made out of components of the vector $\vec{t}$ that dictates the shift $X_{ij} \rightarrow X_{ij} + \vec{t}_{ij}z$.

$$\frac{1}{t_1(t_1 - t_2)(t_2 - t_3) \cdots (t_{n-4} - t_{n-3})t_{n-3}}$$

[SP, Skowronek, Spradlin, Volovich]

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Sum over all **partial triangulations** that contain unshifted chords.
This is the sum over all graphs that can be built from the propagators still available at infinity.

[SP, Skowronek, Spradlin, Volovich]

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Product of **numerators** associated to the various subsurfaces.

[SP, Skowronek, Spradlin, Volovich]

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This is always an amplitude of some theory!

[SP, Skowronek, Spradlin, Volovich]

Properties

The result manifests the following properties

- Factorization: Functions at infinity $\mathcal{A}_n^\infty$ factorize into lower-point functions at infinity.
- Commutativity: The order of taking two subsequent g -vector shifts is immaterial.

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- Factorization: Functions at infinity $\mathcal{A}_n^\infty$ factorize into lower-point functions at infinity.
- Commutativity: The order of taking two subsequent g -vector shifts is immaterial.
- Pion amplitudes: For special g -vector shifts we get pion-scalar amplitudes. For example, consider a base triangulation (X_{13}, X_{14}, X_{15}) and a shift vector $\vec{t} = (1, 0, 0)$. This gives

$$\mathcal{A}_6 \xrightarrow{z \rightarrow \infty} \frac{1}{z^2} \mathcal{A}_6[\pi_1 \pi_2 \phi_3 \phi_4 \phi_5 \phi_6]$$

For another example, consider a shift vector $\vec{t} = (1, 0, 1)$. This gives

$$\mathcal{A}_6 \xrightarrow{z \rightarrow \infty} \frac{1}{z^2} \mathcal{A}_6[\pi_1 \pi_2 \pi_3 \pi_4 \pi_5 \pi_6]$$

[Arkani-Hamed, Cao, Dong, Figueiredo, He]

Applications

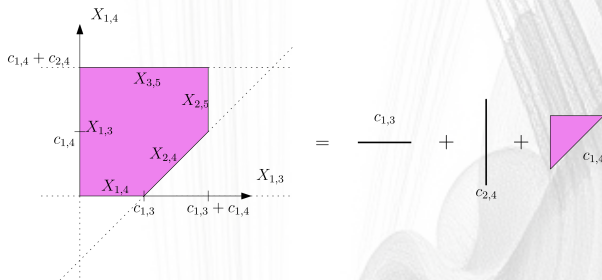
1. **A new positive geometry**: A class of pion-scalar amplitudes are described by products of associahedra
2. **Recursion relations**: These manifest the zero and near-zero splitting properties of $\text{Tr } \phi^3$, NLSM, YMS and special Galileon.
3. **Cosmological wavefunction**: The combinatorics of some graphs that contribute to flat-space coefficients are described by associahedra.

Applications

1. A new positive geometry
2. Recursion relations
3. Cosmological wavefunction

Associahedron

$\text{Tr } \phi^3$ amplitudes are given by the canonical function of the ABHY associahedron. At 5-point:



[Arkani-Hamed, Bai, He, Yan]

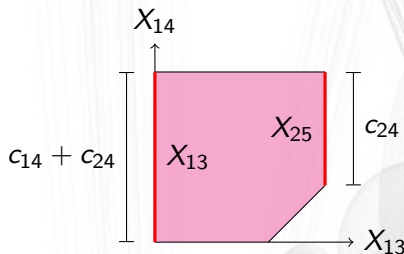
The canonical function is then

$$\frac{1}{X_{14}X_{24}} + \frac{1}{X_{24}X_{25}} + \frac{1}{X_{25}X_{35}} + \frac{1}{X_{13}X_{35}} + \frac{1}{X_{13}X_{14}} = \mathcal{A}_5$$

What happens when we take $X_{14} + z$, $X_{24} + z$, $X_{35} - z$ and $z \rightarrow \infty$?

Projecting the Associahedron

We have **no access to the boundaries** that have been shifted to infinity. All we have is



Since the lowest dimensional boundaries are **no longer vertices**, we have to include the lengths of the **boundary components** in the associated canonical function:

$$\frac{1}{z^2} \left(\frac{c_{14} + c_{24}}{X_{13}} + \frac{c_{24}}{X_{25}} \right)$$

A Special Case

In most cases, like the previous one, we end up with disjoint geometries
i.e. a **union of simplices**.

Can we get a single positive geometry for a special class of deformations?

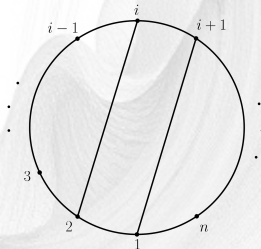
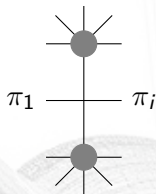
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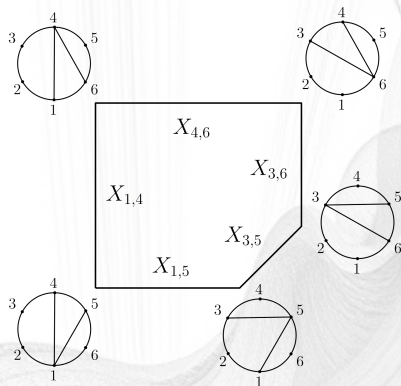
Yes! $\mathcal{A}_n(\pi\phi\cdots\phi\pi\phi\cdots\phi)$

Feynman diagrams give us a good idea of why this is the case:



A New Positive Geometry

Thus a well-defined form on the $(n-1)$ -point ABHY associahedron space gives the n -point $\mathcal{A}_n(\pi\pi\phi\cdots\phi)$ amplitudes.



Meanwhile for the non-adjacent case, we get a product of associahedra.

Applications

1. A new positive geometry
2. Recursion relations
3. Cosmological wavefunction

Recursion Relations

For any g -vector shift, the amplitudes vanish at infinity $\Rightarrow$ one can construct a **recursion relation**, similar to BCFW:

$$\mathcal{A}_n(z=0) = \sum_{ij} \text{Res}_{\hat{X}_{ij}=0} \frac{1}{z} \hat{\mathcal{A}}_n(z)$$

For a specific shift, we get a particularly nice set of recursion relations that manifests **hidden zeros and near-zero splitting**.

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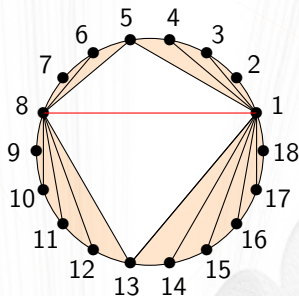
These amplitudes were shown to vanish on the support of

$$\left\{ \begin{array}{l} s_{1m+1} = s_{1m+2} = \cdots = s_{1n-1} = 0 \\ s_{2m+1} = s_{2m+2} = \cdots = s_{2n-1} = 0 \\ \vdots \\ s_{m-1m+1} = \cdots = s_{m-1n-1} = 0 \end{array} \right.$$

and split smoothly when one of these conditions is relaxed.

Shift that Exposes the Zero

We use the shift that corresponds to



It turns out **all but two** of the residues vanish on the support of near-zero kinematics. This leaves us with the following recursion relation:

$$A_n[1, \dots, n] = -\frac{1}{X_{1m+1}} \text{Res}_{z=X_B} \hat{A}_n(z) + \frac{1}{X_{mn}} \text{Res}_{z=X_T} \hat{A}_n(z) = \left(\frac{1}{X_{1m+1}} + \frac{1}{X_{mn}} \right) \text{Res}_{z=X_B} \hat{A}_n(z)$$

which is precisely the **near-zero splitting theorem**.

Multiple Splitting Formulae

On the support of near-zero kinematics:

Theory	Generic	Near-Zero	Explanation?
$\text{Tr } \phi^3$	z^{-2}	z^{-2}	Projective Invariance
NLSM	z^0	z^{-2}	Commutativity of g -vector shifts
YMS	z^0	z^{-2}	?
SGal	z^2	z^{-2}	?

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Additionally, one can also get **higher-order splitting** formulae

$$\begin{aligned}
 A_n^{\phi^3} [1, \dots, n] &\xrightarrow[\{\mathbf{c}_{1,j}, \mathbf{c}_{1,j+1}, \dots, \mathbf{c}_{1,n-1}\} \neq 0]{\mathcal{Z}(X_{1,j})} A^{\phi^3} [1, \dots, j] \left\{ \sum_{k=j}^{n-1} \frac{c_{1,k}}{X_{1,j} X_{j-1,n}} A^{\phi^3} [j-1, \dots, n] \right. \\
 &\left. + \sum_{k=j}^{n-1} \sum_{l=j}^k \frac{c_{1,l}}{X_{1,j} X_{1,k+1} X_{j-1,k+1}} A^{\phi^3} [j-1, \dots, k+1] A^{\phi^3} [k+1, \dots, n, 1] \right\}.
 \end{aligned}$$

[Jones, SP]

Applications

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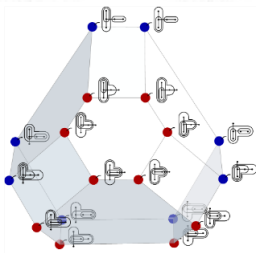
Tubings and Cosmological Wavefunction Coefficients

Observable: cosmological wavefunction graphs (or their integrands) of a conformally coupled scalar in flat space (or de Sitter).

Combinatorics: Given by graph tubings

Relation to $\text{Tr } \phi^3$:

- The wfcs have hidden zeros, near-zero splitting and geometric descriptions.



[De, SP, Pokraka, Spradlin, Volovich]

- Are there corresponding g -vector shifts? Can we get theories with derivatives like NLSM?

Next Questions

- The function at infinity is **fully characterized** by considering the surface description of $\text{Tr } \phi^3$.
- It always **factorizes** and is sometimes a pion amplitude.
- Projecting the associahedron gives a **positive geometry** for $\mathcal{A}_n[\pi\phi \cdots \pi\phi \cdots \phi]$.
- Theories have zeros and split near them if and only if their **residue at infinity vanishes**.
- Many of these properties **extend** to cosmological wavefunction graphs.

Next Questions

- The function at infinity is fully characterized by considering the surface description of $\text{Tr } \phi^3$.
- It always factorizes and is sometimes an amplitude.
- Projecting the associahedron gives a positive geometry for $\mathcal{A}_n[\pi\pi\phi\cdots\phi]$.
- Theories have zeros and split near them if and only if their residue at infinity vanishes.
- Many of these properties extend to cosmological wavefunction graphs.
- Is there a **polytopal** description of NLSM?
- Which other theories have vanishing residues at infinity?
- To what extent do **cosmological observables** exhibit surface descriptions?
- Can we use such properties to fix integrands at **loop-level**?

String Amplitudes

The surface integral formulation of $\text{Tr } \Phi^3$ amplitudes generalizes simply to the **disk integrals of open string theory**.

Can one then understand the effect of g -vector shifts on string amplitudes?

Usual Gross-Mende limit:

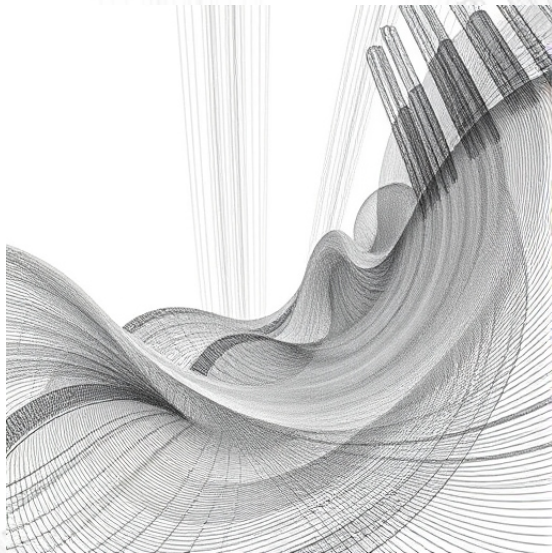
$$\frac{\Gamma(\alpha' s)\Gamma(\alpha' u)}{\Gamma(\alpha' s + \alpha' u)} \xrightarrow{s, u \rightarrow \infty} \sqrt{\frac{2\pi}{\alpha'}} s^{\alpha' s} u^{\alpha' u} t^{-\alpha' t} \sqrt{\frac{s+u}{su}}.$$

A surprising suppression when doing a g -vector shift. All s^s terms vanish, leaving a power law:

$$z^{-1-\alpha' t}.$$

Similar relations occur at 5-point. Investigation is underway at 6-point.

WIP [Arkani-Hamed, Figueiredo, SP, Skowronek]



Thank You